

Understanding quantum physics through simple experiments: from wave-particle duality to Bell's theorem

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Quantum physics, which describes the strange behavior of light and matter at the smallest scales, is one of the most successful descriptions of reality, yet it is notoriously inaccessible. Here we provide an approachable explanation of quantum physics using simple thought experiments that deal with one- and two-particle interference. We derive all relevant quantum predictions using minimal mathematics, without introducing the advanced calculations that are typically used to describe quantum physics. We focus on the two key surprises of quantum physics, namely wave-particle duality, which deals with the counter-intuitive behavior of single quantum particles, and entanglement, which applies to two or more quantum particles and brings out the inherent contradiction between quantum physics and seemingly obvious assumptions regarding the nature of reality. We employ Hardy's version of Bell's theorem to show that so-called local hidden variables are inadequate at explaining the behavior of entangled quantum particles. This means that one either has to give up on hidden variables, i.e. the idea that the outcomes of measurements on quantum particles are determined before an experiment is actually carried out, or one has to relinquish the principle of locality, which requires that no causal influences should be faster than the speed of light. Finally, we describe how these remarkable predictions of quantum physics have been confirmed in experiments. We have successfully used the present approach in a course that is open to all undergraduate students at the University of Calgary, without any prerequisites in mathematics or physics.

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I. INTRODUCTION

Quantum physics, our most successful description for the physics of the tiny, is notoriously hard to understand. The difficulty stems from the conflict between the predictions of quantum physics and common-sense human intuition, which is relatively at ease with the laws of

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classical physics. Experts of quantum physics, such as Einstein, Bohr, Heisenberg, de Broglie, and Feynman, have admitted to being baffled by it. Thus, it might seem utterly hopeless to explain the theory to non-experts. Nevertheless, in this manuscript, we attempt to convey to non-experts the essence of the *strangeness* of quantum mechanics. We convey this strangeness without invoking daunting technical details, but at the same time, without resorting to inaccurate analogies or compromising any essential rigor in treatment.

Now, to be sure, popular culture abounds with accounts of the strangeness of quantum mechanics—jokes, motifs in science fiction, and so on. But the prevalent notion of the strangeness of quantum mechanics is no different from that which one may associate with *any* idea from a field in which one lacks expertise. In addition, because of similarity in language, jargon from quantum mechanics has been misapplied to new-age thought and this can propagate a false sense of mystique and obscurity. Such a feeling is clearly different from the bafflement that a physicist may experience by studying quantum mechanics. The latter results not from a lack of comprehension of the technical ideas, but on the contrary, from a precise understanding both of these technical ideas and of their conflict with commonsense notions. It is this latter sense of strangeness which we attempt to communicate here.

Readers that have encountered (secondary school-level) mathematical concepts of probability, square roots, and imaginary numbers are qualified to grasp our arguments. Nonetheless, for clarity, we include sufficient introductions to these concepts to supplement our physical discussions.

We focus on two key ways in which quantum mechanics is at odds with our intuition about the macroscopic world. The first of these conflicts is the so-called *wave-particle duality*. Through a discussion of simple experiments involving light, we describe how tiny particles seem to possess both wave- and particle-like behavior. That is, some experiments can be explained by thinking of light as composed of particles and others can be explained by treating it as a wave. A more satisfactory explanation of single-particle experiments is provided by quantum physics, which requires us to consider all the different possible paths leading to any given detector. Under certain conditions, these paths can interfere in a manner that is analogous to classical waves like water or sound waves. We show exactly how quantum physics explains the wave-like and the particle-like behavior of single-particle experiments.

We then enter even stranger territory: quantum interference between two or more particles. We approach such two-particle phenomena by first describing *quantum entanglement*, a property that collections of quantum particles can possess. Entangled particles exhibit randomness in their individual measurement outcomes but these outcomes are random in a correlated manner, like synchronized dancers with unpredictable but coordinated moves.

This interplay between correlation and randomness is such that entangled particles cannot be described as individual entities and must be described as one composite entity. Entanglement epitomizes the essential incompatibility of quantum physics with the seemingly-obvious assumptions about the nature of reality. Using our single-particle (example) experiments as building blocks, we piece together an accessible demonstration of this incompatibility, which is the second key revelation of quantum physics.

Despite the strangeness of single-particle interference, its outcomes can also be explained completely by ‘mechanistic’ theories in line with our everyday, classical, experience. In these so-called *hidden-variable theories*, each particle carries information about the eventual outcomes of all measurements that can be performed on the particle, a bit like a hidden “cheat sheet” that the particle can refer to. Einstein, Podolsky and Rosen (EPR) had suggested in 1935 that this approach might allow one to explain all of the apparent strangeness of quantum physics in an intuitive way ([Einstein et al., 1935](#)).

However, in 1964 John Bell famously showed ([Bell, 1964](#)) that when one tries to apply the hidden variable approach to entangled particles, one encounters a major difficulty. Bell showed that so-called local hidden variables can not explain the predictions of quantum physics for entangled particles. One either has to give up on the idea of hidden variables altogether, or one has to accept that these hidden variables have to be “non-local”, i.e., the hidden variables associated with one particle sometimes need to be updated instantaneously based on the behavior of another, possibly very distant, particle. This latter possibility is at odds with the principles of Einstein’s famous theory of special relativity, which imposes a universal speed limit on the rate at which a cause can have an effect, a concept referred to as “locality” in the present context.

It is worth pointing out immediately that despite this dramatic conclusion, entangled particles cannot be used to transmit information faster than at the speed of light. We will see that this important principle is protected by the randomness of the individual measurement outcomes for each particle.

After describing the EPR argument in detail, we piece together concepts from the described one- and two-particle experiments to present the Hardy’s version of Bell’s theorem ([Hardy, 1992](#)). We use this version because it allows us to demonstrate the theorem with minimal mathematics. Moreover, because Hardy’s approach shines a bright spotlight on the sharp contrast between quantum predictions and everyday thinking, it provides a more intuitive understanding of the second key surprise of quantum physics.

We conclude with a brief overview of real-world experiments that have demonstrated the strangeness of quantum physics. Beginning with single-particle experiments that show experimentalists’ technical prowess in generating

and controlling fragile quantum systems, we describe experimental progress up to recent efforts that decisively ruled out local hidden variables by closing all the loopholes in previous experiments that alternate explanations had relied upon. In this discussion, we hope to convey a sense of the extraordinary effort and thoroughness that scientists have put into testing quantum physics.

Our approach, which uses simple experiments to explain quantum principles, was inspired by the popular book of Scarani (2006). Compared to the excellent second book by Scarani *et al.* (2010) and the equally excellent book by Rudolph (2017), our approach relies more on real (or at least realistic) experiments, without introducing more advanced mathematical concepts such as state vectors. Remarkably, it is still possible to derive all the relevant quantum predictions just using the various relevant single-particle and two-particle histories. The expert reader might anticipate correctly that these histories are basically generalized Feynman paths (Derbes, 1996; Feynman *et al.*, 2010). We also recommend articles by Kwiat and Hardy (2000) as well as Christensen and Kwiat (2017) that pedagogically describe Hardy’s theorem.

The spur for developing our approach was an annual undergraduate course, titled “Quantum Mysteries and Paradoxes”, that we have taught for several years at the University of Calgary. This course is open to students from all Faculties without any prerequisites. Our success in communicating the principles of quantum physics to students with a broad range of backgrounds (majoring in the arts and business as well as in the various sciences) leads us to believe that our approach may also be useful in other educational contexts, as well as for motivated individual readers with little background in science or mathematics.

This manuscript is organized as follows. Sec. II introduces single-particle interference and wave-particle duality. Sec. III discusses two-particle interference and entanglement. Sec. IV introduces the Einstein-Podolsky-Rosen argument and the concept of local hidden variables. Sec. V develops Hardy’s version of Bell’s theorem, demonstrating that local hidden variables cannot reproduce the predictions of quantum physics. Sec. VI summarizes the large body of experimental work that has verified these quantum predictions. Section VII summarizes these results and looks towards the future.

II. SINGLE-PARTICLE INTERFERENCE AND WAVE-PARTICLE DUALITY

Here we introduce the first key surprise of quantum physics: that tiny particles display both wave-like and particle-like behavior. To explain this so-called wave-particle duality, we discuss simple experiments (which we name S1 through S6) with single particles of light, i.e. photons. These experiments can be and have been

performed with other tiny particles, such as electrons or neutrons but we chose photons because it is more intuitive to think in terms of light sources, mirrors and detectors.

Experiments S1 and S2 illustrate the particle aspect of single photons, whereas Experiments S3 and S4 reveal their wave aspect. Experiments S5 and S6 help clarify the conditions under which one observes particle-like or wave-like characteristics of single photons.

After a description of the experiments we briefly recap the classical theory of waves and describe how this can explain the outcomes of Experiments S3 and S4. Next we present the unified explanation of all these experiments as provided by quantum physics. This is done through a set of rules which quantum physics uses for predicting the experimental outcomes and which encapsulate two important principles of quantum physics, namely the superposition principle and the principle of indistinguishability. We conclude this section with a surprising quantum phenomenon, referred to as “interaction-free measurement”, that allows one to infer the presence of an object without directly interacting with it, which will play an important role in Hardy’s version of Bell’s theorem (Sec. V).

A. Experiments with Photons

Each of the following experiments involve a single-photon source, some beam-splitters, and some single-photon detectors. A single-photon source fires one photon at a time, ruling out the possibility of two photons being in the setup of the experiment at the same time. A beam-splitter is a device that splits a continuous stream (i.e. a beam) of particles into two equal parts. In an experiment with a beam of light, one can think of a beam-splitter as simply a “semi-transparent mirror”; it reflects half of the beam and transmits the rest of it (see Fig. 1:S1). To observe photons, we use single-photon detectors, i.e., devices that can indicate the presence of a single photon. We will sometimes refer to this indication as a “click” (imagining an audible signal), but in practice the signal is typically electrical. These simple building blocks are put together into six experiments, S1–S6, that enable one to appreciate the particle-like and wave-like properties of photons.

1. Experiment S1

The setup of the first experiment comprises a single-photon source, a beam-splitter, and two detectors, as shown in Fig. 1:S1. We consider a situation in which the single photon source directs photons, one at a time, towards the beam-splitter. We know that a macroscopic beam of photons is split into two equal halves under these conditions, but what will happen to individual photons?

To answer the question, the experiment is conducted

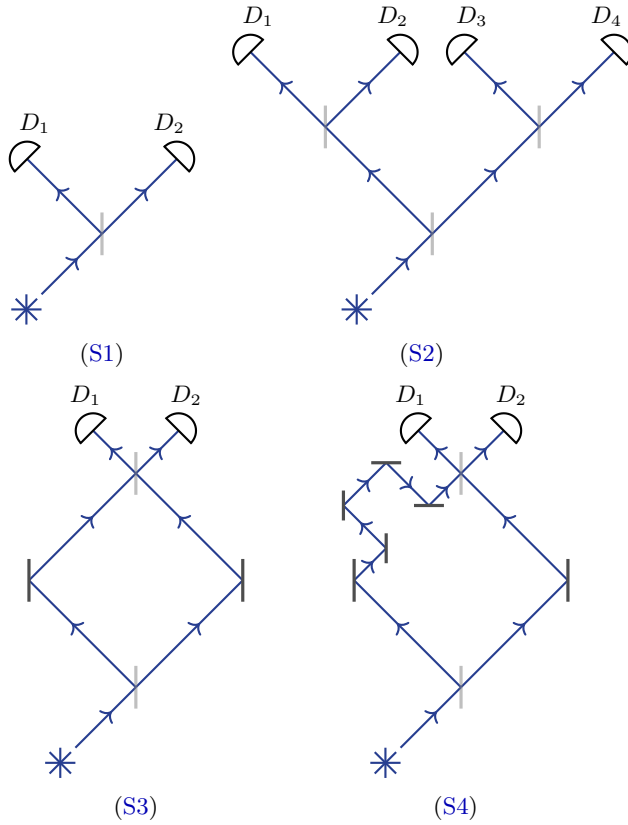


Figure 1: Single-particle interference experiments. (S1) Photons are emitted one by one by a single-photon source and directed towards a beam splitter. Photons that are reflected are detected by detector D_1 , and photons that are transmitted are detected by detector D_2 . (S2) Extension of the previous experiment. Every photon now encounters a second beam splitter before being detected. (S3) Mach-Zehnder interferometer. Mirrors are used to make the two paths emerging from the first beam splitter cross again on a second beam splitter. There are now two possible ways for each photon to arrive at each detector. For example, to arrive at D_2 , the photon could have been transmitted by the first beam splitter and reflected by the second beam splitter or vice versa. (S4) The left path in the Mach-Zehnder interferometer is made longer by inserting four mirrors.

and the detection outcomes are recorded. The first important observation is that for a single photon there is always only a single detection. There is either a click in detector D_1 or a click in detector D_2 , but never in both. It is this observation because of which we can talk about the photon as if it is a particle of light; it cannot be split into two.

The second observation is that the clicks seem to occur randomly. On an average, half of the photons are detected in D_1 and half in D_2 , but there is no discernible sequence to which detector will actually click. This suggests that when a photon reaches a beam-splitter, it seems to make a random choice about which path to take: with probability

$1/2$ it decides to take the path towards D_1 , and with probability $1/2$ it takes the path towards D_2 .

2. Experiment S2

The setup for Experiment S2 is depicted in Fig. 1:S2. To better understand the effect of a beam-splitter on single photons, we consider an experiment with two more beam-splitters in each outgoing path of the first beam-splitter. Where will the photons be detected now?

Based on the previous experiment, we would expect that there will be only one detection for each photon sent. This is indeed what one observes in this experiment as well. Moreover we would expect that the photons will be detected in all four detectors with equal probability of $1/4$, since for each beam-splitter the photon should have an equal probability ($1/2$) of being transmitted or reflected. So, for each detector the probability that the photon reaches it is $1/2 \times 1/2 = 1/4$. This expectation is also confirmed if one performed this experiment: similar to the case of Experiment S1, there is no way to predict which detector will actually click during a given trial, but on average the detectors click with equal probability. This experiment thus reinforces the idea that the photon is a particle that seems to make a random choice at each beam-splitter.

3. Experiment S3

The setup of the third experiment, called a Mach-Zehnder interferometer, is depicted in Fig. 1:S3. The first half of the setup is similar to that of Experiment S1. Then, with the use of two perfect mirrors, we redirect the two outgoing paths of the first beam-splitter so that they meet again. A second beam-splitter is placed at the meeting point. Two single-photon detectors are located at each outgoing path of this second beam-splitter. Again sending in photons one by one, where will they be detected now?

First of all, we again expect only a single detection for each photon sent into the setup, and this is indeed what is observed.

Secondly, based on the experience with the first two experiments, one might expect the photons to be detected with an equal probability of $1/2$ in each detector. On reaching the first beam-splitter, each photon should have a probability of $1/2$ to reflect or to transmit, i.e., to take the left path or the right path. Then at the second beam-splitter, each photon should have an equal probability to be transmitted or reflected, no matter which path it took previously. This should lead to equal probabilities for the two detectors.

In more detail, one could argue as follows. There are two possibilities for the photon to arrive at detector D_1 . It

can take the left path (with probability $1/2$) and then be reflected (with probability $1/2$), giving a total probability of $1/2 \times 1/2 = 1/4$ for this possibility. Or it can take the right path (with probability $1/2$) and then be transmitted (with probability $1/2$), giving a total probability of $1/2 \times 1/2 = 1/4$ for this second possibility. Adding up the two possibilities should give a total probability of $1/4 + 1/4 = 1/2$ for the photon to be detected in D_1 . An analogous argument can be made for detector D_2 .

However, this is not at all what is observed. In fact, *all* the photons are detected by detector D_2 and none by D_1 ! This clearly contradicts the simple model of the photon making random choices at each beam-splitter. Something different must be happening. As we will explain in detail in Sec. II.B, the photons are showing a wave-like character here—even though they are still arriving one by one at the detector D_2 in a particle-like manner! Now let us consider a slightly modified version of the Mach-Zehnder interferometer.

4. Experiment S4

The setup of this experiment is identical to that of Experiment S3, except that the left path has been extended with the help of some (perfect) mirrors (see Fig. 1:S4). What will happen to the percentage of photons detected in D_1 and D_2 as we vary the length of the path extension?

It is not immediately clear why the length of the path would matter. However, the path length has a dramatic effect on the outcome of the experiment: as we increase the path length on the left, detector D_1 starts clicking some of the time, with a probability initially increasing as we lengthen the path. Of course, the probability of detection by D_2 decreases accordingly. At some point, with an extension by a certain length (call it L), we detect no photons at D_2 : all the photons are now detected in D_1 . The detection pattern has reversed from its original form! If we extend the length further, D_2 starts to register clicks again, with greater probability as we increase the length (with a simultaneous decrease in the click probability at D_1). When the extension is $2L$, the original pattern is recovered, and the photons are all detected in D_2 again. For an extension by $3L$, the photons all go to D_1 again, and so on, the pattern reversing for each additional L added to the path.

This behavior is difficult to understand if one tries to imagine the photon as a particle taking just one of the two paths within the interferometer. Considering the beam-splitter that is placed right after the source, it is natural to suppose that half of the photons take the left path and the other half take the right path (based on our observations from Experiment S1). Hence, any change made to one of the paths should change the outcomes of no more than half the photons. Instead what is actually observed is that changing just one of the paths (the left

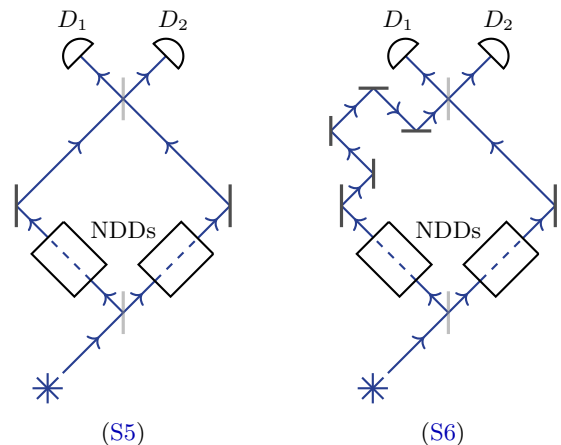


Figure 2: Single-particle experiments with non-destructive detectors (NDDs). (S5) non-destructive detector (NDD) is placed in each path. Depending on which NDD clicks, one knows which path the photon took. (S6) The length of one path is varied while the NDDs are still present in each path.

one in our example) affects the behavior of all of the photons, and not (as one might incorrectly imagine) just the half that are supposed to have taken the left path. Thus, somehow all photons seem to be “aware” of this change in the left path! There is nothing special about the left path, by the way—the same behavior can be observed by changing only the right path. Again all the photons are affected by changing only one of the two possible paths.

5. Experiment S5

Experiments S3 and S4 may leave us wondering: how did all the photons sense a change in one of the paths if only half the photons actually seem to take that path? Did the photons take just one path or both, and if only one path, then which path did they take?

To answer this question we consider yet another modified version of Experiment S3, in which we try to observe the path that the photon takes while keeping the setup otherwise unchanged. Let us assume that we have a device that can measure the presence of a photon in a given path without absorbing the photon or changing its intrinsic properties (for example, its speed or color). We call this device a “non-destructive detector” (NDD). What happens if we redo the experiment with such devices in the paths as depicted in Fig. 2:S5?

First of all, and as expected, for every photon sent into the setup, exactly one NDD will click each time. That is, the photon is detected either path with equal probability so half the photons are observed in the left path and half in the right one.

The mystery of single-particle behavior thickens when we focus on the probabilities of detectors D_1 and D_2 clicking. In contrast to Experiment S3, we no longer observe all the photons in detector D_2 . Instead, now detectors D_1 and D_2 click with equal probability! This experiment actually displays the seemingly obvious outcome that one might have naively expected for Experiment S3 based on Experiments S1 and S2. Now, when their paths are observed, the photons seem to behave like particles making random choices at the beam-splitters again. Merely observing the photons' taken paths somehow prevents the photons from acting like they did in Experiment S3.

One might wonder what would happen if, instead of placing NDDs in both paths, these are placed only in one of the two paths, say the left path. In such a situation, if this NDD clicks, then we know for sure that the photon took the left path between the two beamsplitters. Otherwise, we infer that the photon took the right path. This means that a single NDD suffices to determine which path the photon took, and as a result the detection pattern will be identical to that seen if NDDs are placed in both paths. Going forward, we describe experiments with NDDs in each of the paths, but with the implicit understanding that sometimes we could get the same effect with fewer NDDs.

6. Experiment S6

Another pertinent question for the NDD experiment is ‘what will happen if we change the length of one of the paths?’ That is, what would be the detection pattern for a modified version of Experiment S4 with an NDD in each path as depicted in Fig. 2:S6. Recall that without the NDD there to observe the photons' paths, the detection probabilities were dramatically effected by changing path length. In contrast, when this experiment is performed with the NDD included, then the changing path length turns out to have no effect whatsoever! The photons continue to arrive at detectors D_1 and D_2 with equal probability. This is again what one would have expected based on the straightforward particle model; the presence of the NDD changes the detection outcomes back to the particle-like behavior.

7. Summary: Experiments with Photons and Wave-Particle Duality

Looking back at the first four experiments, there is a clear contrast between Experiments S1 and S2 on the one hand, and Experiments S3 and S4 on the other. In the first two experiments, light seems to show a distinct particle-like behavior, with each photon making a random decision at each beam-splitter on its way towards the detectors. Note that in these two experiments, whenever

a detector clicks, it is possible to unambiguously assign a specific path and imagine that the photon took this path to arrive at this detector. In contrast, in Experiments S3 and S4, each detector click admits two possible paths that the photon could have taken to arrive at that detector. In this case, each photon acts as if it traversed both paths at once, or at least somehow “knew” about both paths. It turns out that these two experiments admit a complete explanation in terms of classical waves, and we go through this explanation in detail in the following section. If we try to measure which of the two possible paths each photon took, like we did in Experiments S5 and S6, then the photons behave like particles once again. That is, we can again predict the outcome by assuming that each photon makes a random choice each time it encounters a beam-splitter.

These six experiments together capture the first key surprise of quantum physics: wave-particle duality. Indeed, not just particles of light, but any quantum particle can display this seemingly-contradictory behavior, which has been tested and probed through decades of experiments (detailed in Sec. VI). In Section II.C, we will take a look at how we can model this behavior and we work our way towards describing how quantum physics can accurately predict these surprising outcomes. Before that, we describe how one can analyze Experiments S3 and S4 using the classical theory of waves.

B. Classical Wave Explanation of Experiments S3 and S4

Here we highlight how the classical theory of waves can explain Experiments S3 and S4. We go through the explanation in detail as some of the features of wave theory, such as the superposition principle will be directly useful in the quantum physics of the following section.

1. Classical Wave Theory

Recall that Experiments S1 and S2 can be understood by modeling the photon as a particle that makes random choices at each beam-splitter. However, this model predicts incorrect outcomes for Experiments S3 and S4. We now show that, on the other hand, the outcomes of Experiments S3 and S4 can be explained if light is described as a wave of electric and magnetic fields. This description of light was suggested as early as 1865 by James Clerk Maxwell (Maxwell, 1865) and has been very successful in explaining a wide variety of phenomena related to light. To distinguish it from the quantum description of light, we refer to this theory as the classical wave theory.

Classical wave theory can explain these experiments by introducing a phenomenon known as interference of waves. Before discussing interference, let us first understand what a wave is. Most waves that we are familiar with

exhibit repetitive patterns, such as water ripples in an otherwise still pond caused by a stone tossed gently from the shore. The oscillations in the water surface have peaks and troughs. These peaks and troughs propagate along the surface of the water away from the point where the stone entered.

When we throw a large stone into the pond, we will notice that the water surface is disturbed more than when we throw a small stone. In general, a larger stone will produce taller peaks and deeper troughs than a smaller stone. From this observation we can define the first important property of a wave: the wave amplitude. Quite simply, the amplitude describes how much a wave disturbs the medium in which it propagates and it is quantified by a positive real number. The amplitude of a water wave is just the height by which the water surface changes when a wave travels on the surface. For classical electromagnetic waves, the amplitude of a wave propagating in a medium is the maximum electric field magnitude that is caused by the wave in that medium. The amplitude of an electromagnetic wave is an important property because the energy of a wave is proportional to the square of its amplitude.

If we now focus on the water surface at one position, we observe that the surface rises when a wave peak arrives. The surface then drops back to its original height, and then drops even more as a trough arrives. Finally, it returns to its original height before the process starts all over again. The complete process of starting from the original height, rising, falling and finally returning to the original height is called a cycle of the wave. The time that it takes for a wave to complete one cycle is called the period. However, if we instead take a picture of the wave at a fixed time, the cycle can also be represented by the distance between subsequent peaks, or equivalently the distance between subsequent troughs. This distance is called the wavelength. The period and wavelength of a wave are related to each other by the speed of the wave. For example, if two waves have the same speed but one has half the wavelength of the other, the shorter wave will have a period half that of the longer wave. The period, wavelength and speed of a wave are three important properties of a wave. For the experiments detailed in this manuscript, we only deal with waves that all have the same period, wavelength and speed¹.

We can now define another important property: the phase of a wave, which describes how much of a cycle the wave has completed at a given position. Each cycle of the wave is divided into 360 degrees. If a wave has just begun

its cycle at a given position then the phase of the wave at that position is zero degrees. After completing half a cycle its phase is 180 degrees, and the phase reaches 360 degrees once the cycle is complete.

We can also define a relative phase between two separate waves that have the same period and speed (and hence, the same wavelength). The two waves are said to be in phase if their peaks and troughs arrive at the same time at a given position. The waves are said to be out of phase if their effect at a given position differs by half of a complete cycle (or 180 degrees). If two waves are out of phase, whenever a peak of the first wave arrives, it is accompanied by a trough of the second wave.

The amplitude and the relative phase can be combined into a single quantity. This quantity is a complex number, denoted a , and is known as the complex amplitude. To explain the magnitude and phase of a wave in terms of the complex amplitude, we first define the imaginary unit as $i = \sqrt{-1}$ so that $i^2 = -1$. The amplitude of the wave is the absolute value² $|a|$ of its complex amplitude a . The phase of a wave is related to the argument of its complex amplitude, which is defined as the angle between the complex number and the real axis in the complex plane. For instance, consider a wave with an amplitude of 1, then the absolute value of the complex amplitude is $|a| = 1$. We consider four illustrative complex amplitudes: $a = 1$, $a = i$, $a = -1$, and $a = -i$. In terms of wave cycles, $a = i$ implies that the wave is shifted by one-quarter of a cycle from $a = 1$. That is, it has a relative phase of 90 degrees. Similarly, $a = -1$ and $a = -i$ are shifted by one-half (180 degrees) and three-quarters (270 degrees) of a cycle respectively. These four cases are illustrated in Fig 3a. See Fig 3b for an example of a wave with complex amplitude $a = i/\sqrt{2}$ and its comparison to a reference $a = 1$ wave³. For simplicity, hereafter we will refer to the complex amplitude as just the amplitude unless clarification is required.

2. Interference of Waves and the Superposition Principle

Now that we have introduced the important properties that define a wave, we are in a position to describe their interference, which is a key concept in understanding not just classical wave theory but also quantum physics. Let us first return to the analogy of the stone tossed into a still pond. This time, instead of a single stone, let us imagine tossing two stones into the pond simultaneously

¹ The speed of light in a vacuum is constant and equal to about 299 792 458 meters per second. Light cannot travel faster than this value, but it can travel slower if it travels through matter. For example, the speed of light in glass is roughly two-thirds of its speed in a vacuum.

² Recall that the absolute value of a complex number $a = x + yi$ is $|a| = \sqrt{x^2 + y^2}$. Thus, the magnitude of $i = 0 + 1i$ is $|i| = 1$. Similarly, we have $|-1| = 1$, from which it follows that $|i| = |-i| = |-1| = |1| = 1$.

³ For an electromagnetic wave, the energy is related to the squared amplitude. Therefore a wave with a complex amplitude of $a = i/\sqrt{2}$ ($|a|^2 = 1/2$) will have half the energy of a wave with $a = 1$.

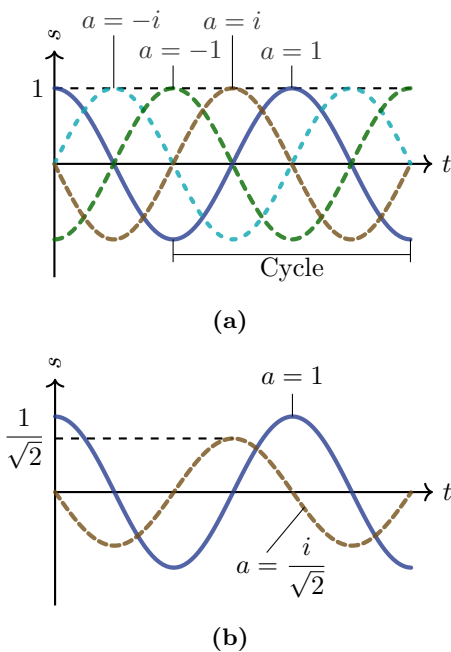


Figure 3: A visual description of amplitude, cycle, and phase. (a) The displacement s plotted against time t for four waves with the same amplitude ($|a| = 1$) and period but different relative phase. The complex amplitude a describes both the amplitude and phase of each wave. With respect to $a = 1$, the waves labeled by $a = i$, $a = -1$, and $a = -i$ are shifted by one-quarter, one-half, and three-quarters of the cycle, respectively. In this case, since the horizontal axis is time, the labeled cycle is the wave period. (b) An example of a wave ($a = i/\sqrt{2}$) with both a different amplitude and phase relative to $a = 1$. The wave $a = i/\sqrt{2}$ is reduced in height and delayed by one-quarter of a cycle compared to $a = 1$. This is what happens for a reflected wave at a beam-splitter.

at separate but nearby locations. As the waves on the water surface propagate outwards from the two stones, they will meet each other to form an intricate pattern. At some positions, the two waves are ‘in phase’, such that peaks and troughs of the two waves will coincide, resulting in a wave with an even taller peak and deeper trough—the waves exhibit constructive interference. At other points, the waves are completely ‘out of phase’ such that the peaks of one wave cancel the troughs of the other—the waves exhibit destructive interference and the water remains still. This pattern of constructive and destructive interference can be explained using the superposition principle:

The superposition principle.— When two or more waves meet, they are superimposed and their amplitudes add together to create a single new wave.

In other words, when two waves meet at a given position, the resulting motion of the surface is described by a third wave whose amplitude equals the sum of the amplitude

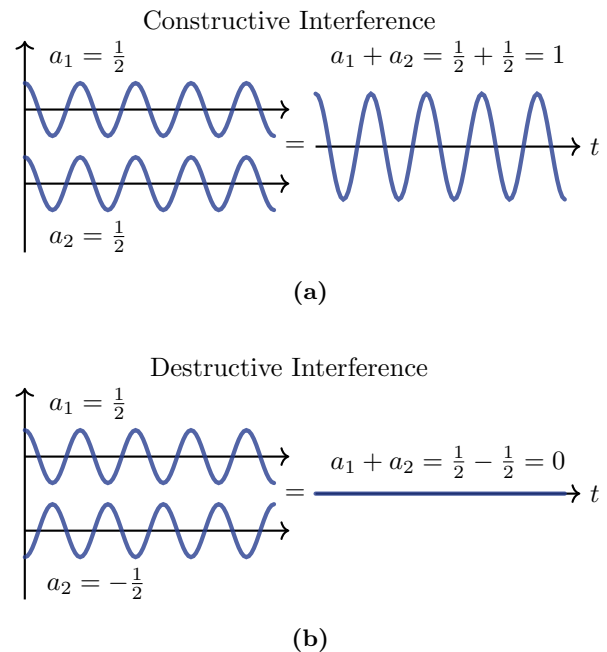


Figure 4: Illustrations of constructive and destructive interference of waves. (a) Two waves of amplitude $a_1 = a_2 = 1/2$ meet and constructively interfere to form a new wave $a_1 + a_2 = 1$. (b) Two waves of amplitude $a_1 = -a_2 = 1/2$ meet and destructively interfere to form a new wave $a_1 + a_2 = 0$.

of the original two waves. This allows us to understand constructive and destructive interference as follows. Let us consider a situation when a wave with an amplitude $a_1 = 1/2$ at a given position meets a wave described by the same amplitude $a_2 = 1/2$ at the same position, as depicted in Fig 4a. This results in a new wave with an amplitude of $a_1 + a_2 = 1$. Thus, the peaks are higher and the troughs are deeper. We say that this new wave results from constructive interference of the original two waves.

The opposite situation occurs when a wave with $a_1 = 1/2$ meets with a wave with $a_2 = -1/2$ at the same position. This results in $a_1 + a_2 = 0$, so the resulting wave has zero amplitude! More generally, when two waves of different amplitude meet such that the peaks of one wave coincide with the troughs of the other, the resultant wave has smaller amplitude (than the wave that originally had larger amplitude), and this is called destructive interference. In the case that the amplitudes of two incoming waves are equal and if their peaks align perfectly with the troughs of the other, then destructive interference could result in the two waves completely canceling each other out, as is depicted in Fig 4b.

We are now equipped with the tools required to appreciate the wave-like behavior of light in Experiments S3 and S4. We start by constructing a model to describe the effect of a beam-splitter on light, assuming that light is an electromagnetic wave. Recall that the energy of an

electromagnetic wave is equal to the square of its amplitude, $|a|^2$. Let us denote the amplitude of the incoming light wave by a and the amplitudes of the light that is outgoing along the two output paths by a_T and a_R , where the subscripts refer either to transmitted (T) or reflected (R) light. Since the energy of the incoming light should be equal to the total energy of the outgoing light⁴, we find that the square $|a|^2$ of the amplitude of the incoming light should be equal to the sum $|a_T|^2 + |a_R|^2$ of squares of the outgoing amplitudes. Furthermore, because the beam-splitter divides the energy of the incoming beam equally among the two outgoing paths, it follows that the energies of the two outgoing beams should be equal to each other and to half of the incoming energy. Hence, the amplitudes of the outgoing beams are equal to each other and smaller than the incoming amplitude by a factor of $1/\sqrt{2}$. Mathematically, we have $|a_T| = |a_R| = |a|/\sqrt{2}$. This solution is valid because the incoming energy $|a|^2$ is the same as the total outgoing energy $|a/\sqrt{2}|^2 + |a/\sqrt{2}|^2 = |a|^2$.

In addition, if we measure the phase of the outgoing light, we can find that the wave reflected by a beam-splitter is shifted by one-quarter of a cycle with respect to the transmitted wave. We can choose the transmitted wave a_T as the reference wave: $a_T = a \times 1/\sqrt{2}$. Then the reflected wave has a phase of i , so that $a_R = a \times i/\sqrt{2}$. From this, we can see that whenever a beam of light is incident on a beam-splitter, the amplitude of the transmitted wave is determined by multiplying the original amplitude by $1/\sqrt{2}$. Similarly, the amplitude of the reflected wave is determined by multiplying the original amplitude by $i/\sqrt{2}$. This reflected wave is illustrated in Fig 3b as compared to the incoming wave with $a = 1$.

Using this model for beam-splitters and mirrors, the results from Experiments S3 and S4 can be explained in terms of classical light waves. We will provide a detailed quantum explanation of these and the other experiments in the following section but for now we focus on the wave-like behavior exhibited by these experiments. In particular, the incoming light in Experiment S3 can be associated with a wave. We can track its wave amplitude in order to predict the detection outcomes. Upon arrival at the first beamsplitter, the incident beam of light is split into two waves that travel through opposite arms of the Mach-Zehnder interferometer. The wave that is reflected at the beam-splitter is shifted in phase by one-quarter of a cycle compared to the transmitted wave. When the two beams arrive at the second beam-splitter, both the waves split yet again into a total of four waves, two of which take the path to D_1 and the other two towards D_2 . The constructive and destructive interference of the

overlapping waves will lead to the observed detection rates.

Specifically, let us consider the two waves that contribute to light arriving at D_1 . The paths that these two waves take are illustrated in Fig 5. The first of these two waves is the one that was reflected at the first beam-splitter and was once again reflected at the second beam-splitter. It acquires two phase shifts of one-quarter of a cycle each, so it is now shifted by one-half of a cycle compared to the reference. The other contribution is from the wave that was transmitted through both beam-splitters, and so it acquires no relative phase shift. This means that the two waves arriving at D_1 have a relative phase of half a cycle (180 degrees), i.e., they are completely out of phase. Hence, when the two waves recombine at detector D_1 , they must superimpose destructively. Likewise, both the waves that recombine at detector D_2 have been once reflected and once transmitted, so they are in phase. Hence, they superimpose constructively. This explains the outcome of Experiment S3, that all the light is detected in one detector D_2 and no light is detected in D_1 .

The difference between reflection and transmission is not the only way to accumulate a relative phase. If one wave is physically delayed with respect to another, it will also acquire a relative phase. In Experiment S4, we observed that adding a path extension of a certain length L in one path switched the outcomes for D_1 and D_2 so that all the light arrived at D_1 . The interference pattern changes because one wave must take slightly longer to travel through the extension and so its arrival at the second beam-splitter is delayed with respect to the wave in the other path.

When a path difference equal to half the wavelength of the incident light is introduced, the wave arriving at the second beam-splitter is delayed by half of a period—it acquired a relative phase of 180 degrees by passing through the path extension. For light arriving at detector D_1 , this additional phase adds to the 180-degree phase acquired by the wave that experienced two reflections. Hence the two waves arriving at D_1 have a total relative phase of 360 degrees (or 0 degrees), and so they are once again in phase and interfere constructively. On the other hand, the two waves arriving at D_2 also gain a relative phase of 180 degrees from the extension. Since they were in phase prior to adding the extension, they are now out of phase and will interfere destructively. Thus, the interference pattern is reversed. In fact, by continuously varying the path length, the model predicts that the interference effect alternates smoothly between constructive and destructive interference. In other words, the light alternates smoothly between being detected entirely at D_1 , to being detected partially at D_1 and partially at D_2 , to being detected entirely at D_2 and then back again. This dependence of light intensity on path length is a signature of interference and the wave-like behavior of light.

⁴ This is in line with an important principle of physics, namely the conservation of energy. Strictly speaking, the incoming and outgoing energies being equal assumes that no light is lost on the way, for example through absorption at the beam-splitter.

So far, the explanation of wave interference in these experiments does not require any mysterious quantum concepts. This interference can be explained purely as a result of classical physics. Although the classical theory of light can explain the detection outcomes of Experiments S3 and S4, it cannot explain the first two experiments.

Recall that in Experiment S1, we observed that detectors D_1 and D_2 will never click at the same time. Indeed, it was observations such as this one that led physicists to realize that there exist quantities of light that cannot be divided further into smaller parts, and can therefore be considered as particles. This particle-like behavior of light, as described first by Einstein in the context of the photoelectric effect, was in-fact among the observations that led to the birth of quantum theory.

In contrast, if we model the incoming single photons as classical waves, then both the detectors will receive light of equal, albeit smaller, amplitude. The smaller amplitude would explain that half the photons are observed in one detector and half in the other but it would also predict that both the detectors should click simultaneously in at least some cases. However, this is not what is observed in actual experiments, in which photons are never split at the beam-splitter but instead take either one path or the other. Furthermore, this wave model also cannot explain why adding NDDs in the arms of a Mach–Zehnder interferometer destroys the interference pattern, like we observed in Experiments S5 and S6.

From analyzing the experiments using classical wave theory, we can now identify exactly why the light in Experiments S3 and S4 are said to exhibit wave-like behavior. However, this theory cannot explain the wave–particle duality of quantum particles. For that, we must turn to quantum physics.

C. Quantum Physics Explanation of Experiments

The duality between wave-like and particle-like properties of quantum particles that we observed in the previous section is quite different from the characteristic behavior of objects that we encounter on a day-to-day basis. Thus, explaining this behavior requires a different way of looking at these experiments. The framework that successfully describes this surprising behavior is given by quantum physics. In this section, we develop a set of basic rules to predict the outcomes of experiments on quantum particles. We conclude the section by applying these rules to explain the outcomes of the single-particle experiments.

1. Histories and Their Associated Wave-Functions

A good starting point is to understand exactly where classical wave theory breaks down in explaining phenomena involving photons or other tiny particles. Recall that

a single photon incident on a beam-splitter does not split into two photons that are then detected simultaneously at the two detectors. Instead, the single photon remains a full single photon and travels either to one detector or the other, but not both. However, classical wave theory makes the assumption that the energy of light is proportional to the squared absolute value of the wave amplitude. This assumption would mean that the beam-splitter, which splits light waves into waves of smaller amplitude, should split a single photon into two photons of smaller energy. This splitting of photons is not observed in the experiment. Hence, this assumption cannot be true at the quantum level.

Quantum physics introduces a radical new way to predict the outcomes of experiments. The quantum approach merges the idea of random choices made by particles with the interference behavior predicted by classical wave theory. This allows quantum physics to make accurate predictions for experiments with quantum particles—capturing the effects of wave-particle duality.

In quantum physics, a wave is associated with the *history* of each particle that results in a given outcome. For single-particle experiments such as Experiments S1 through S6, these single-particle histories are just the possible paths taken by the particle through the apparatus. In other words, each path that the photon could in principle take to arrive at a given detector is a single-particle history that contributes to a detection at that detector. For experiments involving more particles, a history refers to the set of paths taken by all the particles participating in the experiment and we will discuss multi-particle histories in more detail in the next section (Sec. III). Note that ‘history’, here, is not intended to invoke the idea of time but rather the idea of possibilities; we could replace the word history with the word ‘possibility’ and the quantum physics description would still be sensible.

What do these waves and their associated amplitudes represent? Recall that the classical wave theory explanation presented in Sec. II.B.1 envisaged a wave as a physical structure, something that could be directly observed, like a ripple on the surface of a pond. In quantum physics on the other hand, the wave associated with each history represents something less tangible. Quantum physics connects the amplitude of the wave with *probability*. More precisely, the squared absolute value $|a|^2$ of the amplitude of a wave does not represent energy as it did in classical wave theory; rather it represents the probability of observing a particular outcome for a particle or a set of particles. In quantum physics, the complex amplitude is called the wave-function of the quantum particle.

We emphasize that the wave-function is not something that can be directly observed but rather can be seen as a mathematical tool that is used to make quantum predictions. Consider the example of Experiment S1 as explained in the language of wave-functions. The wave that is associated with the incoming particle is

split into two waves each heading towards one of the two detectors. The amplitudes of the outgoing waves are equal to each other and smaller than the amplitude of the incoming wave by a factor of $\sqrt{2}$. This means that the respective probabilities of detecting single photons at the two detectors are equal to each other and half as large as the probability of sending a single photon towards the beam-splitter. The following section presents a complete discussion of single-particle quantum physics in terms of single-particle histories and their associated wave functions.

2. Indistinguishable Histories and the Rules of Single-Particle Quantum Physics

Other than its physical meaning, most of the properties of waves discussed in Sec. II.B.1 are directly applicable to the wave-function. Of particular interest is the possibility of interference. In quantum physics, the wave-functions of different histories taken by a particle can also be added together, and this can result in constructive or destructive interference. However, unlike classical waves, wave-functions that overlap do not always interfere. Instead, the superposition principle can only be applied if the wave-functions are those of fundamentally indistinguishable histories. By fundamentally indistinguishable, we mean that there is no way, even in principle, to determine which history was taken by the quantum particles to reach the detectors. In other words, photon histories for a particular detection outcome interfere with each other only if it is not possible to determine which history the photons actually took on their way to the detectors.

The interference of indistinguishable histories leads us to an important principle in quantum physics.

The indistinguishability principle.—interference occurs if and only if two or more fundamentally indistinguishable histories contribute toward the same detection outcome.

As an example, consider Experiment S3. This experiment is done in such a way that the single-particle histories are fundamentally indistinguishable, i.e., no physical system in the setup can gather any information about which path the photon took on its way to the detector. Thus the indistinguishability principle mandates that interference occurs between the two single-particle histories that lead to the same detector. In the specific case of a detection at D_1 , the history that comprises two reflections is fundamentally indistinguishable from the path that involves two transmissions, and both of these histories contribute towards a detection in D_1 . Hence, the waves associated with these two histories interfere and, as detailed in the next section (Sec. II.C.3), give us the detection probabilities that we observe.

What about the detection probabilities observed in Ex-

periment S5? Recall that placing an NDD in one of the paths destroys the interference. This is because the NDD provides information on which path the photon took. As a consequence, the histories are now distinguishable. In other words, the histories are no longer fundamentally indistinguishable. If there is any way whatsoever to infer which path the photon took, then the indistinguishability principle entails that the amplitudes from different wave-functions cannot be summed, even if those wave-functions represent the same detection outcome. For these distinguishable histories, we must sum the probabilities for each wave-function, rather than the wave-functions themselves, to obtain the final detection probability. We illustrate this concept with an example as follows.

In Experiments S5 and S6, the two histories for each detection are distinguishable. Hence, the two histories act independently, with each history contributing one quarter probability to the photon detection in the given detector (each history passes through two beam-splitters). Summing the probabilities of $1/4$ from each of the two histories contributing to each detection outcome, we obtain the correct detection probability of $1/2$ at D_1 and $1/2$ at D_2 .

Other than the idea of interference between indistinguishable histories, there is one more important difference to consider when moving from classical wave theory to the quantum physics of wave-functions. In classical theory, light can split and travel along two paths but a single photon described by quantum physics cannot. When a single photon arrives at a beam-splitter, instead of choosing a single path, quantum physics describes it as proceeding through the apparatus in a superposition of both paths or both single-particle histories. This superposition of histories can continue until a path measurement is made, at which point one of the two possible outcomes will result, and we say that the superposition has collapsed. This collapse precludes the detection of the particle in the other detector. It is this collapse that leads to the particle-like behavior of some quantum experiments.

Consider again the example of Experiments S5 and S6. As a photon leaves the first beam-splitter it enters into a superposition of two paths. However, as the photon passes through the NDD, a measurement is made on the photon's path and the indistinguishability between the histories is lost. After the NDD, the photon is no longer in a superposition of the two histories so the photon must have definitely taken one of the two paths, which eventually leads to the two paths do not interfere but rather contribute to detection probabilities independently. Thus, concepts of indistinguishability and superposition are closely related and fundamental for explaining many quantum phenomena.

Let us consolidate what we have learned so far about using quantum physics to predict outcomes of experiments involving light. We gather the operating principles described above into the following basic rules for single-

particle quantum experiments.

Rules of single-particle quantum physics.—

1. To predict the probability of detecting a particle at a specific detector, consider all the histories of the particle that result in the particle arriving at the detector.
2. Each history of a single quantum particle is associated with a wave and its corresponding complex amplitude a .
3. The amplitude of a history is multiplied by a $1/\sqrt{2}$ factor each time the particle is transmitted through a beam-splitter and by a factor of $i/\sqrt{2}$ each time it is reflected at a beam-splitter. Mirrors leave the amplitude unchanged.
4. A history's amplitude can incur a phase shift relative to another history if it involves a particle taking a longer path compared to the same particle in another history.
5. If two or more fundamentally indistinguishable histories lead to the same detection outcome, then the amplitudes of those histories must be summed to obtain the final detection amplitude.
6. The probability (Pr) of detecting the particle in the chosen detector is proportional to the squared absolute value of the detection amplitude: $\text{Pr} = |a|^2$. If more than one distinguishable history leads to the same detection, then their probabilities are summed directly.
7. Observing a particle in one path precludes observing it in any other path.

Equipped with these basic rules, we can now provide an explanation of the single-particle experiments described above.

3. Explanation of Experiments S1 through S6

We now explain the results of Experiments S1 through S6. For each experiment, we predict the pattern of detections using the following procedure: Firstly, for each outcome, identify each history (i.e., path) that contributes to the outcome. Calculate the associated amplitudes from the incoming amplitude using the rule for beam-splitters and phase shifts. Next, apply the indistinguishability principle, i.e., sum the amplitudes from fundamentally indistinguishable paths. Finally, compute the probability of each outcome by taking the squared absolute value of the amplitude and sum probabilities from different distinguishable paths. Let us see this procedure in action.

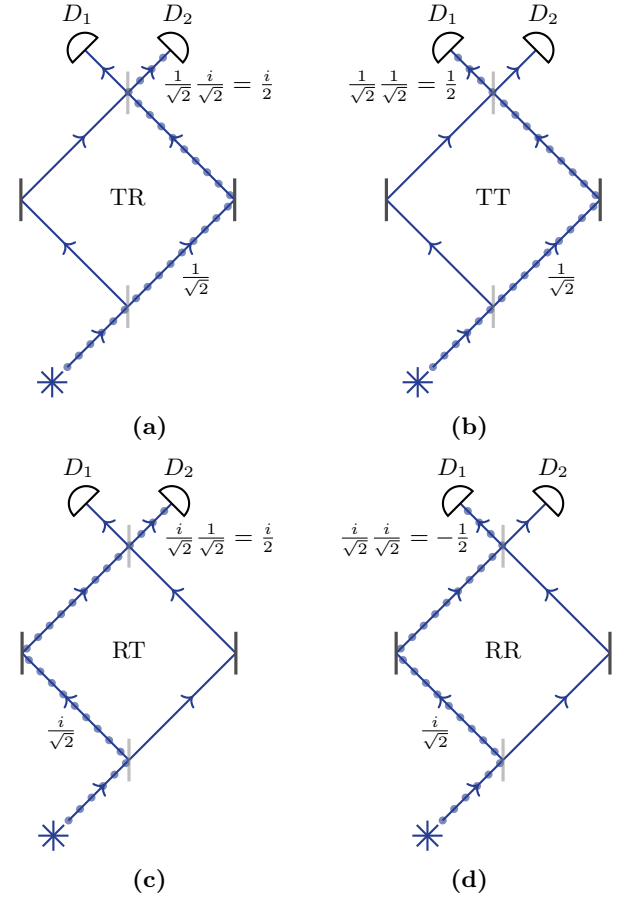


Figure 5: The four possible histories for the single photon in Experiment S3. Subfigures (a) and (c) show the two possible histories for outcome D_2 whereas (b) and (d) show the two possible histories for outcome D_1 . The labels T and R represent ‘transmitted’ and ‘reflected’, respectively. For example, path TR corresponds to a photon transmitted through the first beam-splitter and reflected by the second. The displayed amplitudes can be calculated using the rules described in the main text.

For each outcome of Experiment S1, there is only one contributing history. For a detection at D_1 , this history corresponds to a reflection off of the beam-splitter (Rule 1). Likewise, for a detection at D_2 , this history corresponds to a transmission through the beam-splitter (Rule 1). When a single photon is emitted from the source with unit probability, the incoming amplitude of the photon prior to the beam-splitter is $a = 1$ (Rule 2). The amplitude of the reflected history is $a(D_1) = 1 \times i/\sqrt{2}$ whereas the amplitude of transmitted history is $a(D_2) = 1 \times 1/\sqrt{2}$ (Rule 3). Since there is only one history contributing to each detection outcome, there are no longer or shorter paths that might have incurred a phase shift (Rule 4). Moreover, since only one history contributes, we need not sum over any amplitudes (Rule 5). Hence, the probability of detecting the single photon at D_1 is $\text{Pr}(D_1) = |1/\sqrt{2}|^2 = 1/2$ (Rule 6).

Likewise, the probability of detecting the photon at D_2 is $\Pr(D_2) = |i/\sqrt{2}|^2 = 1/2$ (Rule 6). In this way, one can also show that this procedure recovers the result expected for Experiment S2. Thus, we can correctly predict the detection probabilities in the first two experiments. Furthermore, Rule 7 allows us to predict the correct detection pattern, including the fact that the detectors do not click simultaneously. That is, if the photon is observed in one of the paths, then the photon cannot be present in the other path. More simply, only one of the two (or four) detectors of Experiment S1 (or S2) will click for each single photon input into the experimental setup.

For Experiment S3, there are two possible histories for each of the two detection outcomes. This is because the photon could take either the left or the right path on its way to either detector. Let us label each possible history by a sequence of letters ‘T’ and ‘R’ for transmissions and reflections, respectively, at the beam-splitters. For outcome D_1 , the two possible histories are TT and RR. That is, to end up in D_1 , the photon is either transmitted twice or reflected twice (Rule 1). For outcome D_2 , the two possible histories are TR or RT (Rule 1). These histories are highlighted in Fig 5.

Rule 2 stipulates that each history is associated with an amplitude. As in S1 and S2, the incoming amplitude is $a = 1$. Then the amplitude of each history is determined by following Rule 3 for the beam-splitters and mirrors:

$$\begin{aligned} a(\text{TR}) &= \frac{1}{\sqrt{2}} \times \frac{i}{\sqrt{2}} = \frac{i}{2}, \\ a(\text{TT}) &= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2}, \\ a(\text{RT}) &= \frac{i}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{i}{2}, \\ a(\text{RR}) &= \frac{i}{\sqrt{2}} \times \frac{i}{\sqrt{2}} = -\frac{1}{2}. \end{aligned}$$

Rule 4 is not applicable here because we assume that there is no path-length difference between different histories of the same outcome. In addition, the histories RR and TT are fundamentally indistinguishable since there is no information available that indicates which path the single photons actually took. Similarly, the histories RT and TR are fundamentally indistinguishable. Then by Rule 5 we have the following detection amplitudes:

$$\begin{aligned} a(D_1) &= a(\text{TT}) + a(\text{RR}) = \frac{1}{2} - \frac{1}{2} = 0, \\ a(D_2) &= a(\text{TR}) + a(\text{RT}) = \frac{i}{2} + \frac{i}{2} = i. \end{aligned}$$

Next, we use Rule 6 to predict the detection probabilities:

$$\begin{aligned} \Pr(D_1) &= |a(D_1)|^2 = |0|^2 = 0, \\ \Pr(D_2) &= |a(D_2)|^2 = |i|^2 = 1. \end{aligned}$$

This gives us the correct probability of detecting photons in D_1 and D_2 respectively.

To explain Experiment S4, we must account for the relative phase introduced by the path extension added to the left arm of the apparatus as informed by Rule 4. In particular, the amplitudes for the histories that include a reflection at the first beam-splitter must accumulate an additional half-cycle phase (180 degrees) due to the extension, so we have:

$$\begin{aligned} a(\text{RT}) &= \frac{i}{\sqrt{2}} \times -1 \times \frac{1}{\sqrt{2}} = -\frac{i}{2}, \\ a(\text{RR}) &= \frac{i}{\sqrt{2}} \times -1 \times \frac{i}{\sqrt{2}} = \frac{1}{2}. \end{aligned}$$

On the other hand, the amplitudes for the histories that include a transmission at the first beam-splitter remain the same as in Experiment S3.

$$\begin{aligned} a(\text{TR}) &= \frac{1}{\sqrt{2}} \times \frac{i}{\sqrt{2}} = \frac{i}{2}, \\ a(\text{TT}) &= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2}. \end{aligned}$$

We can now see that the summed detection amplitudes are reversed for the two detectors:

$$\begin{aligned} a(D_1) &= a(\text{TT}) + a(\text{RR}) = \frac{1}{2} + \frac{1}{2} = 1, \\ a(D_2) &= a(\text{TR}) + a(\text{RT}) = \frac{i}{2} - \frac{i}{2} = 0. \end{aligned}$$

Thus the probabilities are $\Pr(D_1) = 1$ and $\Pr(D_2) = 0$, and the constructive and destructive interference pattern is swapped.

These same rules also enable an unambiguous explanation of Experiments S5 and S6. If the arms of the Mach-Zehnder interferometer are monitored using an NDD, then it is known which path each photon takes. In other word, the contributing histories are now distinguishable. In this case, we can predict the correct outcome probabilities using Rule 6.

Recall that detection D_1 resulted from histories involving two transmissions (TT) or two reflections (RR). The presence of the NDDs makes histories TT and RR distinguishable based on which NDD detects the photon. As the contributing paths are distinguishable, the probabilities are calculated for the distinguishable outcomes individually and added as described by Rule 6:

$$\Pr(D_1) = |a(\text{TT})|^2 + |a(\text{RR})|^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2},$$

Similarly detections in D_2 result from the histories TR and RT, which are now distinguishable because of the presence of the NDDs. Thus the probability of detection in D_2 is obtained by summing the individual probabilities of detection for the cases TR and RT according to

$$\Pr(D_2) = |a(\text{TR})|^2 + |a(\text{RT})|^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}.$$

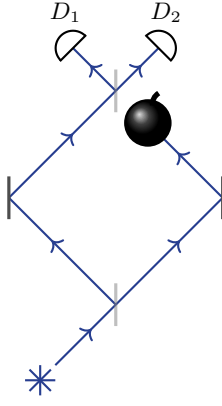


Figure 6: Interaction-free measurement setup. A successful measurement can detect the presence of the bomb—which has blocked the right path—without any photons from the source hitting it.

Applying Rule 6 gives us the correct $(1/2)$ detection probabilities for each detection.

Notice that the additional path length of Experiment S6 has no effect on the detection probabilities. This is because the eventual detection probabilities do not depend on the complex probability amplitude, but depend only on the individual probabilities [i.e., $|a(TT)|^2$, $|a(TR)|^2$, $|a(RT)|^2$ and $|a(RR)|^2$], which are independent of any phase acquired in the paths. This leads to identical detection probabilities for the two detectors in Experiments S5 and S6, and no interference is observed in either experiment. In summary, the presence of the NDDs destroys the interference and causes the single photons to exhibit particle-like behavior similar to Experiments S1 and S2.

Let us summarize the quantum description of single-particle experiments. We have demonstrated that a few simple principles of quantum physics can completely explain the particle-like and wave-like behavior of light in many experiments. This enables us to make correct predictions for the probabilities of detecting quantum particles, even if the physical meaning of the principles may still seem a bit mysterious.

There are more mysteries and surprises ahead once we consider two or more quantum particles. Before moving on to the case of two-particle interference, we will introduce the concept of interaction-free measurement, which is important for understanding Hardy’s version of Bell’s theorem later in the manuscript.

D. Interaction-Free Measurement

Can one infer the presence of something without ever having interacted with it? It turns out that single-particle quantum experiments allow for exactly such a possibility.

Usually when we think of measurements using light, we imagine reflecting light off of the object that is being observed and detecting the reflected light. The so-called “interaction-free” measurement is a way to use light to detect the presence of an object based on single-photon interference without the light physically interacting with the object.

The setup for illustrating interaction-free measurement is very similar to the setup of Experiment S3—the Mach-Zehnder interferometer. We have one single-photon source, two beam-splitters, and two mirrors. However, the right path is blocked midway by an obstacle. To heighten the drama, quantum physicists like to assume this object is a “bomb” that will explode if a photon strikes it (see Fig. 6). Can we detect the presence of the bomb in the setup without any explosion occurring?

First, we suppose that there is no bomb in the setup. In this case, the setup is exactly the same as Experiment S3. This allows us to predict that an incident single photon will always be detected by detector D_2 .

On the other hand, in the case that there is actually a bomb in the right path, the probability of detecting a photon at each detector will change as follows. At the first beam-splitter, an incident photon will choose the right or left path with an equal probability and thus there is a $1/2$ probability that the photon will hit the bomb and make it explode. These are unsuccessful trials from the point of view of interaction-free measurement. However, there is a $1/2$ probability for the photon to take the left path and arrive at the second beam-splitter. Here, the photon either takes the path leading to D_1 or the one leading to D_2 with equal likelihood. Thus for an incident photon, either the bomb explodes with $1/2$ probability or one of the two detectors will detect the photon each with an equal probability of $1/4$.

So far, we have seen that if there is no bomb in the setup, then an incident photon always arrives at D_2 ; but if there is a bomb, then it could arrive at D_1 or D_2 . If a single photon is detected in D_2 , then we cannot say if the bomb was present because there is a chance for a photon to arrive at detector D_2 in both cases. However, this is completely different if a photon arrives at D_1 . In this case, we can infer that the bomb must be present in the setup, because otherwise all of the photons would interfere and none would take the path towards D_1 . Hence, for a single incident photon, there is a $1/4$ probability that we can infer the presence of the bomb in the setup without making the bomb explode. In this way, quantum physics allows us to detect an object using light without the light ever having interacted with the object.

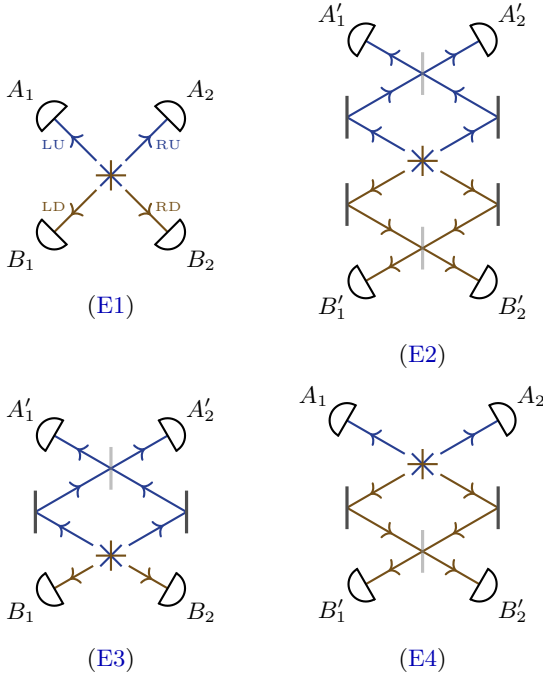


Figure 7: Two-particle interference experiments. (E1) Direct measurement of the photons when emitted from the entangled-photon source, depicted as the colored star in the center. The source emits two photons back to back. (E2) the setup for observing two particle interference by merging paths for both Alice and Bob (E3) and (E4) If only one of Alice and Bob merge the paths of the interferometer before detection, then no interference is observed.

E. Summary of Single-Particle Experiments Quantum Physics

Single quantum particles exhibit wave-particle duality, a mixture of particle-like and wave-like behavior. Quantum physics explains single-particle behavior by firstly attaching a complex-valued amplitude to each history, then demanding that amplitudes from fundamentally indistinguishable paths be summed, and finally by deriving probabilities of detection as the squared absolute value of the total amplitude. This strange single-particle behavior is exemplified by the possibility of performing an interaction-free measurement. We now turn our attention to the strange behavior of two quantum particles with a focus on entanglement, which encapsulates the second key surprise of quantum physics.

III. TWO-PARTICLE INTERFERENCE AND ENTANGLEMENT

So far, we have only discussed single-particle experiments, which demonstrate wave-particle duality. In the early days of quantum physics, this strange duality was a

source of much debate. Nonetheless, as quantum physics matured, and found more and more success in explaining and predicting the outcomes of ever-improving experiments, wave-particle duality became an accepted feature of quantum physics.

Making the leap from one to two quantum particles exposes a second, and key, surprise of quantum physics, that of entanglement. Entanglement is a property of two or more quantum particles that prohibits the description of each particle as an individual entity. The properties of entangled particles depend on each other, but cannot be known before being observed. That is, a random result is obtained from a measurement of an individual particle. This interplay between mutual dependence, or correlation, and individual randomness is the signature of entanglement. As we discuss below, these properties make the concept of entanglement counter-intuitive, as well as open to being misunderstood.

Entanglement has many surprising consequences. For example, if two particles are entangled, then the outcome of an experiment that is performed on one of the two particles can instantaneously alter the expected outcome of an experiment that is performed on the second, separately (and potentially, distantly) located, particle. One might imagine that this could be exploited for sending messages faster than light, for example by performing measurement at one location and then looking at the changes in the expected outcomes at a distant position. Sending messages faster than light, also called signaling, would violate the special theory of relativity, which mandates no messages can be sent faster than the speed of light. Einstein, who founded the theory of relativity, derisively referred to such instantaneous action as “spooky action at a distance”, and considered this seemingly-absurd prediction as evidence that quantum physics had to be an incomplete description of reality, as we will describe in more detail in the next section (Sec. IV). In fact, as we explain in Sec. III.B, quantum physics conforms with locality because it is not possible to use entanglement to send a signal, either faster-than-light or otherwise.

To explain the concept of entanglement, we now present some simple two-particle experiments that can be understood by generalizing the ideas that we introduced in Sec. II. We also present the quantum physics explanation of these experiments using rules similar to the ones introduced for single particles. We conclude this section with a short description of why entanglement cannot be used to send messages faster than the speed of light.

A. Two-Particle Experiments and their Quantum Explanation

The experiments involve the same equipment that we have encountered before, with one addition: a source of entangled pairs of photons. The source emits two photons,

one upwards and one downwards, each either into the left path or the right path. The two possible paths for the upwards going photon are labeled *LU* (Left-Up) and *RU* (Right-Up), whereas those for the downward going photon are *LD* (Left-Down) and *RD* (Right-Down) as shown in Fig. 7:E1. The photons are always emitted in pairs. Two photons from an emitted pair travel in opposite directions, which could either be *LU* and *RD*, or be *RU* and *LD* with equal likelihood.

The two photons that are emitted by the source exhibit the basic signature of entanglement: the paths that the photons take are simultaneously correlated and random. The paths that the photons take are correlated because if the upwards going photon takes the *LU* path, then the other certainly takes the *RD* path. Likewise for the case of one photon taking the *RU* path, which implies that the other photon takes the *LD* path. Moreover, the path that any individual photon takes is not known beforehand, it is completely random.

Entanglement thus describes a special kind of quantum superposition of two or more particles. But what exactly is in a superposition? Two or more particles could be in superpositions over different possible histories, where a history now refers to the collection of the paths taken by all the particles in the experiment.

While the aforementioned source of photon pairs is not typically constructed in real life, sources with analogous behavior are well-developed and are routinely used in experiments. We discuss this in Sec. VI. It is convenient to imagine that the experiments of Fig. 7 are being performed by two experimenters, named Alice and Bob. Alice is located above the source and receives the upward going photon, while Bob is located below the source, receiving the downward going photon. The path of the photon traveling towards Alice is depicted in blue and the detectors that are used by Alice are denoted with an ‘*A*’. Similarly, the path of the photon traveling towards Bob is depicted in brown, and his detectors are denoted with a ‘*B*’. Our use of colors is not meant to imply anything about the color of the light in the actual photons, but merely to emphasize that there are now two different photons, in contrast to the experiments that we have encountered so far (Sec. II.A). Now we examine five different experiments that are performed by Alice and Bob.

1. Experiment E1

First we consider a simple situation in which both Alice and Bob have placed detectors directly in the paths of the photons. Thus, Alice can detect if the upward going photon is traveling in the left path *LU* via detector *A*₁ or the right path *RU* via detector *A*₂. Likewise, Bob can distinguish between the downward going photon travelling in the left path *LD* if detector *B*₁ clicks or the right path *RD* if *B*₂ clicks. This setup is depicted in Fig. 7:E1.

Now we can ask the question, “What is the detection pattern for Alice’s and Bob’s detectors?” That is, how often does Alice detect the photons in *A*₁ and *A*₂ detectors, and how often does Bob detect photons in *B*₁ and *B*₂? More importantly, is there some dependence or correlation between Alice’s detection and Bob’s?

These questions can be answered promptly based on our description of the entangled photon source. Alice’s left detector *A*₁ will click in half the cases and the same for the right detector *A*₂. Similarly Bob’s photon detection events will be split half and half into the two detectors *B*₁ and *B*₂. The outcomes of Alice’s and Bob’s detections are correlated: if *A*₁ clicks, then *B*₂ will surely click. Similarly, if *A*₂ clicks, then so will *B*₁. In other words, the detection events (*A*₁, *B*₂) and (*A*₂, *B*₁) both happen with probability 1/2 whereas the events (*A*₁, *B*₁) and (*A*₂, *B*₂) happen with zero probability.

This detection pattern can be thought to result from the detectors picking out one out of the two possible histories for the photon pairs. As the source only emits photons in the *LU–RD* and *RU–LD* directions, these two pairs of directions are the only two histories possible for the photon pairs. If Alice’s photon is detected in *A*₁, then this outcome is explained by only one out of the two histories, i.e., the *LU–RD* history, which means that Bob’s photon will be detected in *B*₂.

Let us note that according to quantum physics it is completely undetermined which of the two possibilities, *LU–RD* or *RU–LD*, will occur in any given experiment. This is only decided when the measurement is made. Yet, as soon as the upper particle is detected in *A*₁, then we know that the lower particle will definitely be detected in *B*₂, and similarly for *A*₂ and *B*₁. This observation is closely related to the “spooky action at a distance”, which was disturbing to Einstein, among others. We will come back to this point in more detail in Sec. IV.

2. Experiment E2 and the Rules of Many-Particle Quantum Physics

To see how interference manifests itself in the two-particle case, let us now imagine a different setup, which we depict in Fig. 7:E2. In this setup, the upwards going paths are made to meet at a beam-splitter before Alice detects the photons. Similarly, both of Bob’s paths are made to meet at a beam-splitter before the detection is performed. This seemingly small addition completely changes the pattern of detector clicks. The resulting pattern can be explained as arising from two-particle interference, which follows the same basic principles as single-particle interference, but in a somewhat more complex way, as we now explain.

In this two-particle experiment, we are interested in Alice’s and Bob’s detection probabilities and correlations. In other words, what are the probabilities of four dis-

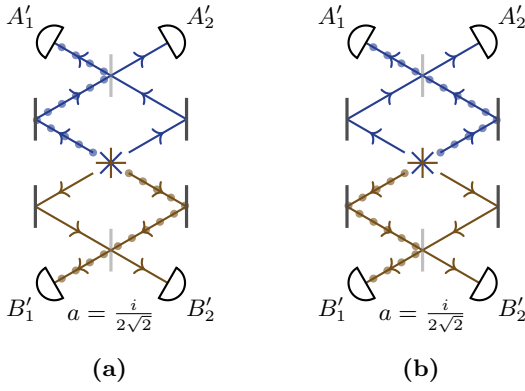


Figure 8: The two histories for outcome (A'_1, B'_1) detection in E2. These are the two indistinguishable histories that contribute to a detection pattern (A'_1, B'_1) in Experiment E2. The two histories contribute a probability amplitude $i/2$ and interfere constructively, thus giving a probability $1/2$ of both detectors (A'_1, B'_1) clicking.

tinct detection events: (A'_1, B'_1) , (A'_1, B'_2) , (A'_2, B'_1) and (A'_2, B'_2) ? We have introduced primes to the detector labels to indicate that these detectors are measuring a different physical property now because the paths now meet at the beam-splitter before arriving at the detector.

To compute the detection probabilities via quantum physics, we can exploit rules similar to those of Sec. II.C. For the many-particle case, we are interested in the probabilities of observing a specific detection outcome, i.e., of observing some specific detectors clicking. As in the single-particle case we assign an amplitude to each history that contributes to the given detection pattern. Furthermore as in the single-particle case, two or more histories will interfere only if they are fundamentally indistinguishable.

Thus, the set of seven rules for single-particle interference are modified to the following for many-particle interference:

Rules of many-particle quantum physics.—

1. To predict the probability of a specific detection outcome, consider the histories of all the particles that result in the particles arriving at the detectors specified by the outcome.
2. Each history of the quantum particles is associated with a wave and its corresponding complex amplitude a .
3. The amplitude of a history is multiplied by a $1/\sqrt{2}$ factor each time an involved particle is transmitted through a beam-splitter and by a factor of $i/\sqrt{2}$ each time it is reflected at a beam-splitter. Mirrors leave the amplitude unchanged.
4. A history's amplitude can incur a phase shift relative to another history if it involves a particle taking

a longer path compared to the same particle in another history.

5. If two or more fundamentally indistinguishable histories lead to the same detection outcome, then the amplitudes of those histories must be summed to obtain the final detection amplitude.
6. The probability (Pr) of observing the given detection outcome is proportional to the squared absolute value of the detection amplitude: $\text{Pr} = |a|^2$. If more than one distinguishable history leads to the same detection outcome, then their probabilities are summed directly.
7. Observing a particle in one path precludes observing it in any other path.

It is each to check that these rules include the single-particle version as a special case.

Note that Rule 2 requires us to specify the probability amplitude of each history depending on the chosen source. The application of this rule was more straightforward, as each history started with amplitude $a = 1$ but we need to be more careful in the two-particle case. In the current experiments, we are using a source of entangled photons. We can assign probability amplitudes to each history based on the fact the photon pairs are only emitted in the $LU-RD$ or $LD-RU$ directions with equal likelihood and never in the other two pairs of directions. Hence, we will assume that the entangled source contributes a factor of $1/\sqrt{2}$ to the amplitude for the histories in which photon pairs are emitted in the $LU-RD$ or $LD-RU$ directions, and it contributes a factor of zero for histories that involve photons emitted in the $LU-RU$ and $LD-RD$ directions. The fact that we are assigning amplitudes to these two histories implies that they are fundamentally indistinguishable unless detectors are placed in their paths.

Note that it would also be consistent with the probabilities mentioned above to, for example, assign an amplitude of $1/\sqrt{2}$ to $LU-RD$ and an amplitude of $-1/\sqrt{2}$ to $LD-RU$, since the probabilities are given by the square of the absolute value of the amplitudes. However, this would nevertheless be a physically different source because the histories would interfere in a different way. The particles would still be entangled in this case, but they would be entangled in a different way. Here we choose the positive value of the amplitude for simplicity.

We now use these rules to calculate the probabilities of different detection outcomes. First, let us consider the outcome (A'_1, B'_1) . Following Rule 1, observe that there are two indistinguishable histories leading to this outcome and we depict these in the two parts of Fig. 8. The first history is the case in which Alice's photon is emitted in direction LU and reflected by her beam-splitter, while Bob's photon is emitted in RD and transmitted by his

beam-splitter as depicted in Fig. 8a. Now, we assign an amplitude to this history (Rule 2). The amplitude for this history is computed by multiplying the three factors arising respectively from the source, from the transmission in Alice's lab, and from the reflection in Bob's lab (Rule 3). Thus we have

$$\begin{aligned} a(LU_A, R_A; RD_B, T_B) &= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times \frac{i}{\sqrt{2}} \\ &= \frac{i}{2\sqrt{2}}. \end{aligned}$$

The other possible history involves Alice's photon being emitted along RU and transmitted at the beam-splitter while Bob's photon is emitted along LD and reflected at the beam-splitter shown in Fig. 8b. Because this history includes contributions from one transmission, one reflection and the factor from the signal, the resultant amplitude for this history is the same as the previous one:

$$a(RU_A, T_A; LD_B, R_B) = \frac{i}{2\sqrt{2}}.$$

The amplitudes for the two potential histories in which the photon pairs were emitted in the directions $LU-RU$ and $LD-RD$ are zero, since the source always produces photons going in opposite directions.

For entangled particles, the two histories described above are fundamentally indistinguishable. We have to add the probability amplitudes resulting from these indistinguishable histories to obtain the total probability amplitude for the given detection outcome (Rule 5). Thus, the probability amplitude for the detection outcome (A'_1, B'_1) is

$$\begin{aligned} a(A'_1, B'_1) &= a(LU_A, R_A; RD_B, T_B) + a(RU_A, T_A; LD_B, R_B) \\ &= \frac{i}{\sqrt{2}}. \end{aligned}$$

Squaring this probability amplitude gives us the probability of the detection outcome A'_1, B'_1 as (Rule 6)

$$\Pr(A'_1, B'_1) = |a(A'_1, B'_1)|^2 = \left| \frac{i}{\sqrt{2}} \right|^2 = \frac{1}{2}.$$

Thus, the probability of detector A'_1 clicking in Alice's lab and B'_1 clicking in Bob's lab for a given pair of photons is one half. Since the amplitudes for the two histories were in phase (i.e., had the same complex phase), this outcome is a result of constructive interference of the histories.

Now let us consider the detection outcome (A'_1, B'_2) . Without going through all of the details as above, there are once again two indistinguishable histories leading to this outcome. Either the particles are emitted in $LU-RD$ and both reflected, or they are emitted in $LD-RU$ and both transmitted. These histories respectively have amplitudes of $\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}}$ and $\frac{1}{\sqrt{2}} \times \frac{i}{\sqrt{2}} \times \frac{i}{\sqrt{2}} = -\frac{1}{2\sqrt{2}}$. Adding these amplitudes together, we see that these two histories

Outcome	Contributing history	Amplitude	Total Prob.	
(A'_1, B'_1)	$LU_A, R_A; RD_B, T_B$	$\frac{1}{2\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
	$RU_A, T_A; LD_B, R_B$	$\frac{1}{2\sqrt{2}}$		
(A'_1, B'_2)	$LU_A, R_A; RD_B, R_B$	$\frac{-1}{2\sqrt{2}}$	0	0
	$RU_A, T_A; LD_B, T_B$	$\frac{1}{2\sqrt{2}}$		
(A'_2, B'_1)	$RU_A, R_A; LD_B, R_B$	$\frac{-1}{2\sqrt{2}}$	0	0
	$LU_A, T_A; RD_B, T_B$	$\frac{1}{2\sqrt{2}}$		
(A'_2, B'_2)	$RU_A, R_A; LD_B, T_B$	$\frac{1}{2\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
	$LU_A, T_A; RD_B, R_B$	$\frac{1}{2\sqrt{2}}$		

Table I: Probabilities and amplitudes for the different possible detection outcomes of experiment E2. Of the four possible outcomes, only two have nonzero probability of occurrence. The remaining two suffer from destructive interference between their individual histories.

destructively interfere, leading to a total amplitude (and probability) of zero. Thus, the event in which Alice's detector A'_1 clicks concurrently to Bob's B'_2 has zero probability.

The remaining two cases (A'_2, B'_2) and (A'_2, B'_1) are similar to the two cases described above. Hence, we can easily see that the respective probabilities of these paths are $1/2$ and 0 . These results are summarized in Table I.

From Rule 7, we know that if a photon is detected in A'_1 , then it cannot be detected in A'_2 . This means that in half the cases A'_1 - B'_1 outcome is observed and in the other half, A'_2 - B'_2 but both outcomes will never be observed simultaneously. In other words, two photons in the setup will only ever give two detector clicks.

We can say that Experiment E2 demonstrates two-particle interference because, out of the four possible detection outcomes, only two outcomes undergo constructive interference and occur with nonzero probability. The other two outcomes suffer from a destructive interference of their respective histories and have zero probability of occurrence. In the respects described thus far, two-particle interference is similar to single-particle interference. However, the cases of one- and two-particle interference are also different in some important respects.

The fundamental difference between one- and two-particle interference is apparent when we consider the experiment from the perspective of the individual experimenters. For both Alice or Bob individually, their half of the system looks a lot like a Mach-Zehnder interferometer (as described in Sec. II), with the difference that now there is an entangled source where the first beam-splitter was placed in the Mach-Zehnder interferometer. Recall that if the two paths of a Mach-Zehnder interferometer are balanced, then all the photons are detected in only one of the two detectors. In contrast, Alice's detector A'_1

clicks with 50% probability, and the same for A'_2 . In other words, if Alice is not aware of Bob's outcomes she sees a completely random detection pattern on her detectors A'_1 and A'_2 . Bob observes the same phenomenon in his lab. It is not apparent from Alice's or Bob's perspectives individually that any interference is happening whatsoever, since both of their individual outcomes are equally likely to occur.

Furthermore, a distinct signature of single-particle interference was that changing the path lengths in one arm of a Mach-Zehnder interferometer changed the detection probabilities. Recall that for zero path length difference, all the photons were detected in the bright detector, but as one of the paths was extended, the photon detections would be split between the two detectors and eventually, for a certain path difference, all the photons would be detected in only the dark detector. Let us consider what happens in a two-particle case, i.e., when Alice extends one of the arms of her interferometer. What are the probabilities of each detector clicking now? One can verify by simple calculations that it is impossible for Alice to affect the single-particle probabilities by using an extension. Thus, no matter how long or short the extension is, the probability of each of her detectors clicking remains fixed at $\frac{1}{2}$. In this aspect as well, the interference that is happening here is fundamentally different from single-particle interference. However, one can also show that, while the individual probabilities for Alice and Bob do not change, the joint probabilities listed in Table II, in fact, do change when the length of the paths is no longer the same. All of these points show that two-particle interference is distinct from the single-particle case. In other words, it is the histories of the complete two-photon system that are interfering, not the histories for either photon individually, and the interference manifests in the two-particle detections (i.e., the joint detection probabilities), not in the detection probabilities for individual particles. This ensures that the two-particle interference cannot be used for faster-than-light signaling as we detail in Sec. III.B.

Note that, just as for Experiment E1, the detection pattern of the complete Experiment E2 again shows the dual signatures of entanglement: correlation and randomness. If detector A'_1 clicks, then B'_1 must click as well. Similarly, if A'_2 clicks, then B'_2 must click as well. Thus, the clicks of the detectors are perfectly correlated. Furthermore, the outcomes of measurements performed in one lab are completely random if viewed in isolation.

3. Experiment E3

We saw in section II that single-particle interference only occurs if there are at least two indistinguishable histories that the particle could take to arrive at the given detector. Similarly, two-particle interference can only happen if there are at least two indistinguishable histories.

This can be illustrated by considering a slightly modified experiment that we depict in Fig. 7:E3. In this experiment, Alice's paths meet at a beam-splitter before detection, while Bob measures the photons directly. Alice's detectors use the primed notation introduced previously because she is using a beam-splitter, whereas Bob is not using a beam-splitter, so we do not use primes to denote his detectors. In this setting, what are the probabilities of each of the four possible pairs of detector clicks? That is, what are the probabilities of the pairs (A'_1, B_1) , (A'_1, B_2) , (A'_2, B_1) and (A'_2, B_2) of detectors clicking?

We can easily check that these probabilities are all equal to $\frac{1}{4}$. This is because only one history contributes to each of the possible pairs of detection outcomes. Each of these histories picks up a factor of $1/\sqrt{2}$ from the source and either a factor of $1/\sqrt{2}$ or $i/\sqrt{2}$ from the beam-splitter, thus giving a total probability of $1/4$.

For concreteness, let us consider the history that leads to detections of the photon pairs in the detectors A'_1 and B_2 . This detection pattern sees contribution from only one history, in which the photons are emitted in the $LU-RD$ directions, Bob's detector B_2 detects Bob's photons, and Alice's photon reflects off from her beam-splitter into detector A'_1 . This gives us a probability amplitude of $1/\sqrt{2} \times 1/\sqrt{2} = 1/2$, which means that there is probability $|1/2|^2 = 1/4$ of detecting photons in the detectors A'_1 and B_2 . A similar analysis gives the same probability for each of the other three detection patterns.

Observe that in this experiment, only one history contributes to each outcome of the experiment. This means that there are no collaborating or competing histories, and so there is no constructive or destructive interference. As a result, all the pairs of detection are equally likely, which is what we see from the above calculations.

4. Experiment E4

What happens instead if Alice detects her photons directly but Bob places a beam-splitter in the paths of his photons before detecting them? This situation (see Fig. 7:E4) is analogous to the previous Experiment E3 with Alice and Bob having changed their roles. In this case too, each detection outcome sees contributions from one and only one history, and each pair (A_1, B'_1) , (A_1, B'_2) , (A_2, B'_1) and (A_2, B'_2) of detection outcomes are observed with equal probability. Similar to the case of Experiment E3, this experiment does not display any interference.

The joint probabilities in Experiments E3 and E4, in both of which no interference takes place, are thus very different compared to the joint probabilities for Experiment E1 or E2 (see Table I). However, in each of these experiments, the individual probabilities on each side are still $1/2$ for each detector. This again highlights how two-particle interference is different from the single-particle

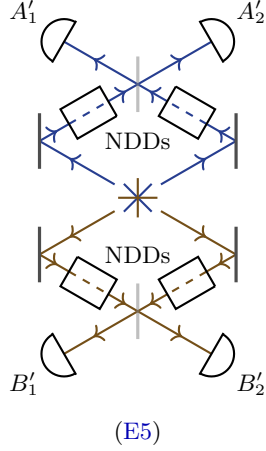


Figure 9: Two-particle interference experiment with non-destructive detectors (NDDs). A non-destructive detector (NDD) is placed in each path of the setup for Experiment (E2). Depending on which NDDs click, one knows which path each photon took.

case.

5. Experiment E5

Two-particle interference, like single-particle interference, can take place only between indistinguishable histories. Let us now think about an experiment that highlights the role of indistinguishability in the two-particle case. We consider a modified version of Experiment E2 as depicted in Fig. 9:E5. Analogous to Experiments S5 and S6, NDDs are placed in the paths of the two particles. Now, what is the pattern of Alice's and Bob's detections and what are the correlations that they observe?

The addition of NDDs in the paths makes the different histories distinguishable. Let us focus on the probability of observing a simultaneous (A'_1, B'_1) detection. The two histories that contribute to this detection, i.e., $LU_A, R_A; RD_B, T_B$ and $RU_A, T_A; LD_B, R_B$ (similar to the highlighted histories of Fig. 8) are now distinguishable. This means that their amplitudes are no longer added together but instead we must follow Rule 6, which suggests that we calculate the individual probabilities of detection from these histories and add them together to obtain the actual detection probabilities.

Specifically, the two histories begin with a $1/\sqrt{2}$ amplitude because of the nature of the source. Both the histories involve one transmission and one reflection, which change the amplitude to $1/\sqrt{2} \times 1/\sqrt{2} \times i/\sqrt{2} = i/\sqrt{8}$. Thus, both histories contribute a probability of $|i/\sqrt{8}|^2 = 1/8$ and these probabilities are added together $1/8 + 1/8 = 1/4$ to obtain the final $1/4$ probability of (A'_1, B'_1) detection. We can check that each of the three other detection patterns (A'_2, B'_2) , (A'_2, B'_1) and (A'_1, B'_2)

have the same $1/4$ th likelihood of occurrence.

This experiment clarifies that indistinguishability is an important prerequisite for two-particle interference and entanglement. Thus, if indistinguishability is removed by adding NDDs to the paths, then Alice's and Bob's detections are still random but no longer display any correlations. This means that in the absence of indistinguishability, the second signature of entanglement is lost.

B. A Note on Signaling, Correlations in Everyday Life

At this point, we are ready to revisit the issue of signaling. Why is Alice not able to use these setups to communicate with Bob faster than the speed of light? The answer lies in the fact that there is nothing that Alice can do locally to influence Bob's probabilities of each detector clicking from his perspective; all Bob ever sees is a string of totally random clicks with detectors B_1 and B_2 clicking randomly with equal likelihood. Thus, there is no way for Alice to use these setups to send a message of any kind, let alone faster than light. In fact, it turns out that entanglement alone can never be used to send messages; some other ingredient is always necessary (such as a telephone line, which manifestly will not allow for fast-than-light messaging).

However, there still seems to be a connection between the two particles that we have not explained so far. In Experiments E1 and E2, how can they give outcomes that are both perfectly correlated, yet apparently completely random? In everyday life, we also often experience correlations, but we can usually find reasons for them. For example, let's consider two people, Alice and Bob, who own shirts of two colors, blue and red. If we saw them wear the same color day after day (sometimes blue, sometimes red, but always the same for the two of them), we would strongly suspect that they are either coordinating their choices somehow, or that they are following a pattern that is known to both of them (say blue when it is sunny in the morning and red when it is not).

This leads to the question what is actually happening for the quantum particles. Is there really some kind of "spooky action at a distance", as quantum physics seems to suggest, and as we briefly described when talking about Experiment E1? Or is there some yet-to-be-discovered simple explanation for the correlations, as in our example with the shirts? In 1935, Einstein, Podolsky and Rosen published an influential paper (Einstein *et al.*, 1935) that argued in favor of the latter conclusion. Now we describe their argument in detail.

IV. THE EINSTEIN–PODOLSKY–ROSEN ARGUMENT AND LOCAL HIDDEN VARIABLES

The EPR argument (Einstein *et al.*, 1935) questions the completeness of quantum theory. The basis of EPR’s argument is the concept of *elements of reality*, which we first describe before presenting their reasoning in detail.

According to EPR, if the outcome of an experiment is known with certainty, then this outcome must correspond to some element of reality in the theory. More specifically, if the value of an experimental outcome can be predicted with certainty in a given situation, then the value definitely corresponds to an element of reality. These experimental outcomes are thought to be real in the sense that they *really have* specific values, even if no measurement is made to ascertain them. Later in his life, Einstein likened this notion of ‘elements of reality’ to the reality of the existence of the moon when no one is looking. Quoting physicist Abraham Pais,

We often discussed his notions on objective reality. I recall that during one walk Einstein suddenly stopped, turned to me and asked whether I really believed that the moon exists only when I look at it. (Pais, 1979)

EPR define a complete physical theory as one that contains a counterpart to each element of reality. According to EPR, a theory is satisfactory if it is complete and correct, in which case the correctness is judged by the degree of agreement between theory and experiment. EPR do not question the correctness of quantum physics, but they argue that it is not *complete* in the sense that it does not contain counterparts to all elements of reality.

The EPR argument relies on an experiment similar to that described in the previous section, in which two particles are shot from an entangled source and measured by two distant experimenters. Although the exact setting that EPR considered was somewhat different, the reasoning can be translated meaningfully into the language of Experiments E1–E4.

A. EPR Argument for Two-Photon Experiments

What are the elements of reality in the two-photon experiments? These are the outcomes of the two different kinds of measurements that can be performed, i.e., detecting photons with and without their paths meeting on the beam-splitter. For concreteness, we focus on the measurements performed by Alice on the upward going photons. Consider the outcome of a Alice’s direct measurement of the photon, i.e., without the paths meeting at a beam-splitter as in Experiments E1 and E4. The outcome of this measurement can be inferred with certainty if Bob measures his photon directly as he did in Experiment E1. According to EPR, this means that some

element of reality must exist that contains information about this outcome.

Next we consider the outcome of Alice’s measurement of her photon after the paths meeting at a beam-splitter. This is the measurement that she had performed in Experiments E2 and E3. As for the case of the direct measurement, the outcome of this measurement performed by Alice can be known with certainty if Bob measures his photon after they meet at a beam-splitter as was done in Experiment E2. Thus, there must be another element of reality that contains information about Alice’s measurement of the photons after they meet at a beam-splitter. If the outcomes of both of Alice’s measurements correspond to elements of reality, then according to EPR, any complete physical theory must contain a description of these outcomes. EPR show that quantum physics does not allow for such a description, and is hence claim that it is not a complete theory.

We can see that quantum physics does not allow for a simultaneous prediction of Alice’s two measurements by considering all four experiments E1–E4 together. First, consider Experiments E1 and E3, in which Bob measures the same quantity, i.e., the location of the photon without placing a beam-splitter in his paths. This allows Bob to infer Alice’s outcome in those instances when she measures the photons directly as in Experiment E1, i.e., when she has not placed a beam-splitter in her paths. However, Bob’s direct measurement of the photons gives no information about Alice’s outcomes in those cases when Alice places a beam-splitter in her photons’ path. This is clear from the outcome of E3, in which Alice’s detections were split randomly between her two detectors. Likewise, we can see that if Bob measures his photon after placing a beam-splitter in his paths (Experiments E2 and E4), then he can infer the outcome of Alice’s measurement if she too had placed a beam-splitter, but not of her direct measurement of the photons.

Because Bob can either measure his photon directly or measure them after the paths meeting, yet cannot perform both measurements at the same time, he can infer the results of only one of Alice’s measurements. Thus, even though both of Alice’s measurements correspond to elements of reality, quantum physics does not allow a simultaneous prediction of these two measurements. As quantum physics does not contain counter-parts to all elements of reality, EPR concluded that quantum physics is not a complete description of reality

EPR end their article with a belief that a complete description of reality does indeed exist even though quantum physics does not provide this description. Thus, the missing ingredient in quantum physics, according to EPR, is the information about eventual measurement outcomes for all elements of reality.

B. EPR Argument and Local Hidden Variables

The modern approach to thinking about the EPR argument and related concepts is in terms of the so-called *local hidden variables*. Local hidden variables are analogous to EPR’s elements of reality but there are subtle differences, which we will get to in Section V.B. Here we present the EPR in the language of local hidden variables.

To explain the outcomes of all measurements that can be performed on a particles, one might imagine that each particle carries a cheat sheet, or a list of instructions, with information about measurement outcomes. These instructions would be the elements of reality sought by EPR. Because these instructions are not a part of quantum physics, and are thus inaccessible to us at least for the moment, they are referred to as local hidden variables in modern discourse.

The *local* part of “local hidden variables” refers to the principle that the physical properties of one particle should not be able to change instantaneously depending on anything done to some other distant particle. This notion of locality is central to Einstein’s theory of relativity, which implies that information cannot travel faster than the speed of light. Let us emphasize that this principle is consistent with all the available experimental evidence to date, including experiments on quantum entanglement.

The EPR argument is based on the assumption of locality because Alice’s elements of reality are thought to be independent of what measurements Bob performs in his lab and vice versa. Assuming that the theory of relativity is correct, locality can be “enforced” by, for example, putting Alice and Bob in distant enough laboratories, such that there isn’t enough time for information to travel from Bob’s lab to Alice’s lab, for instance about which measurement that Bob has performed on his photon, before Alice performs hers. (This point has important consequences for experiments as we describe in Sec. VI.E.)

The question then is: can the outcomes of Experiments E1–E4 be explained using local hidden variables? To explore this question, let’s consider a situation in which Alice and Bob can choose whether to use a beam-splitter or not. Furthermore, our explanation should be able to account for a situation in which they are able to defer their measurement choices until after the source has already emitted its pair of photons. In other words, Alice and Bob can choose whether or not to place a beam-splitter in their paths before detecting the photons, but after the photon pairs have left the source.

Thus, to explain Experiments E1–E4 via local hidden variables, the upward-going photon must leave the source carrying information about its detection outcome if Alice measures using detectors A_1 and A_2 (without the beam-splitter), and also the outcome if A'_1 and A'_2 are used instead (with the beam-splitter). Similarly, the downward-going photon must contain information about its detection outcome for detectors B_1 – B_2 , and also for B'_1 – B'_2 . The

Alice		Bob	
Performed measurements:		Performed measurements:	
A_1/A_2	A'_1/A'_2	B_1/B_2	B'_1/B'_2
A_1	A'_1	B_1	B'_1
A_1	A'_1	B_1	B'_2
A_1	A'_2	B_1	B'_1
A_1	A'_2	B_1	B'_2
A_2	A'_1	B_2	B'_1
A_2	A'_1	B_2	B'_2
A_2	A'_2	B_2	B'_1
A_2	A'_2	B_2	B'_2
A_1	A'_1	B_2	B'_2
A_2	A'_1	B_1	B'_2
A_1	A'_2	B_2	B'_1
A_2	A'_2	B_1	B'_1
A_1	A'_1	B_2	B'_1
A_1	A'_2	B_2	B'_2
A_2	A'_1	B_1	B'_1
A_2	A'_2	B_1	B'_2

Table II: Possible assignment of local hidden variables to explain experiments E1–E4: The sixteen rows of this table list all possible assignments of the local hidden variables for Alice’s and Bob’s photons as they leave the source. The first eight rows are ruled out because experiment E1 shows that detectors A_1 and B_1 never click simultaneously and likewise for detectors A_2 and B_2 . The next four rows are ruled out from the results of Experiment E2, which does not see any simultaneous A'_1 and B'_2 clicks or simultaneous A'_2 and B'_1 clicks. The remaining four rows are valid because they do not contradict the individual results of the four experiments. In fact, if the values in the bottom four rows are assigned randomly with equal likelihood, we can explain the statistics observed in the four experiments.

outcome information that one photon contains can depend on the information that the other photon leaves with. Indeed, the information a photon carries about its destiny may even be allowed to change when the photon is in transit; however, locality forbids such a change to depend on anything done to the other photon, for instance on which measurements are performed on the other photon.

Thus, we can imagine that the source writes two symbols in the instruction list of the upward-going photon: one that determines the outcome of the A_1 – A_2 detection (i.e. A_1 or A_2), and one for the A'_1 – A'_2 detection. Analogously, the source must also write one unprimed symbol (i.e. B_1 or B_2) and one primed symbol (i.e. B'_1 or B'_2) in the instruction list for Bob’s photon. These four symbols are the local hidden variables of the two photons.

Thus, our search for a local hidden variable explanation of Experiments E1–E4 turns into the question of assigning values to the two symbols, one each carried by the two

photon pairs leaving the source. Of course, these values need to be assigned in a way that is consistent with the outcomes of Alice’s and Bob’s detections with and without their paths meeting at a beam-splitter.

Since each of the four symbols can be assigned to one out of two different outcomes, there are a total of $2 \times 2 \times 2 \times 2 = 16$ possible combinations of values that can be assigned to the two particles. We tabulate these values in the sixteen rows of Table II.

From the results of our experiments, we know that some of these combinations are not permitted. For instance, from Experiment E1, we know that A_1 and B_1 must never occur simultaneously. This eliminates the combinations in the first eight rows of Table II as these assignments would have resulted in simultaneous A_1 – B_1 clicks.

Additionally, from Experiment E2, we know that A'_1 and B'_2 never click for photons from the same pair and neither do A'_2 and B'_1 . This eliminates the next four rows of the table, leaving us with only four permissible combinations, i.e., the last four rows.

You may convince yourself that, if one assumes that these last four rows all have equal probability of $1/4$, then one can also explain the results of Experiments E3 and E4. We conclude that if the source simply produces pairs of particles corresponding to one of these four combinations, with equal probability for each of the four, it completely explains the individual outcomes of the four Experiments E1–E4. This seems to support the vision of EPR, even if we may not know yet how to find the local hidden variables for each individual pair of particles.

But does this mean that local hidden variable theories can adequately replace quantum mechanics in explaining all experiments? Einstein continued his quest for such a local hidden variable explanation of all quantum phenomena until the end of his life. This quest was in fact doomed to fail, as shown (after Einstein’s death) by John Bell in his celebrated 1964 paper (Bell, 1964). While local hidden variables can match the outcomes of quantum physics in some cases, as the ones we have discussed so far, they predict radically different outcomes for certain other experiments. In the next section, we turn our attention to an experiment in which no assignment of local hidden variables is consistent with the predictions of quantum physics.

V. EPR, BELL AND HARDY

We are now ready to experience the second, and perhaps strangest, surprise of quantum physics. The story of this surprise began with the EPR paper in which the authors argued that quantum physics is incomplete as a description of reality. Recall that EPR came to this conclusion by making intuitive assumptions about the nature of reality. Although the authors were questioning the completeness of quantum physics under their assump-

tions, they did not question the accuracy of either the assumptions or of quantum physics.

Without realizing, EPR had taken a key step towards uncovering a deep contradiction between quantum physics and two seemingly obvious assumptions about the universe, namely realism (or, equivalently, the existence of hidden variables) and locality. It was John Bell who first realized, to his own surprise, that quantum physics was incompatible with these assumptions. Thus, either quantum physics or one of two cherished assumptions of physics needed to give way.

In his celebrated 1964 paper, Bell formalized the EPR assumptions in terms of local hidden variables (Bell, 1964). Using this formalism, he discovered a contradiction between quantum physics on the one hand and the existence of local hidden variables on the other. Specifically, Bell showed that quantum predictions regarding a certain experimental setup measurably differed from those obtained if local hidden variables were assumed to determine the outcomes of all measurements. This difference could be tested experimentally and opened the way for resolving the question raised by EPR one way or the other. In fact, Bell’s test may be performed by using a two-photon interference experiment similar to that of Fig. E2. Specifically, the test involves varying the length of the paths that photon travels (akin to those of single-particle Experiment S4) and measuring correlations as we discussed in the previous section.

In 1992, Lucien Hardy proposed a simplified, and perhaps more striking, version of the Bell test (Hardy, 1992). Understanding Hardy’s version is less demanding than Bell’s original version because it requires simpler mathematics. Furthermore, Hardy’s version is amenable to a simple explanation using only the concepts of single-particle and two-particle interference introduced in the previous sections, without having to introduce more mathematically advanced concepts such as state vectors or inequalities. Here we follow Hardy’s logic, cast into the interference language of Secs. II and III, to convey the clash between quantum physics and our intuition about nature.

A. The Setting for Hardy’s Paradox

Hardy’s paradox is based on the four experiments depicted in Fig. 10. We refer to these experiments as H1, H2, H3 and H4. The experiments rely on the interference of an electron and its antiparticle⁵, a positron, each in a Mach-Zehnder interferometer, which we introduced in

⁵ Each fundamental particle is associated with an antiparticle, which has the same mass but opposite charge. When a particle and an antiparticle meet, they annihilate and emit radiation in the form of possibly high-energy photons.

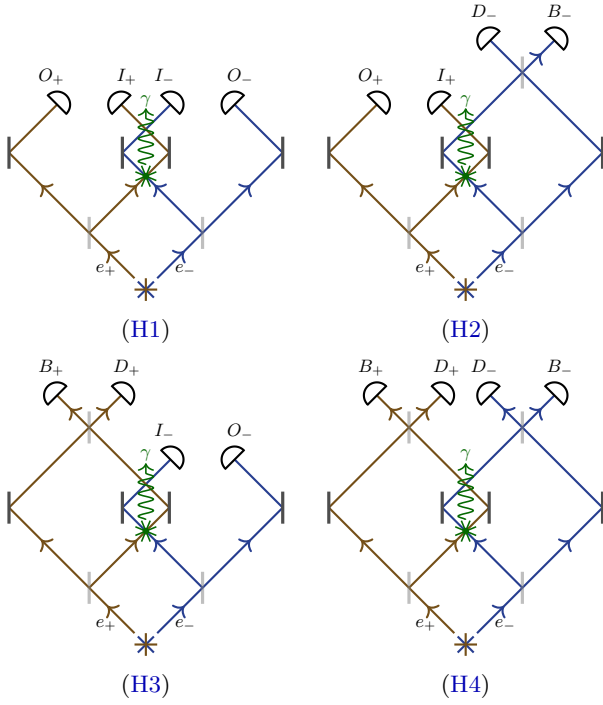


Figure 10: A depiction of Hardy’s thought experiment. The star at the bottom of the figure depicts a source of electron-positron pairs. Of a generated pair, the electron, depicted as e_- , is directed towards the right beamsplitter and the positron, depicted as e_+ , to the left beamsplitter. Each of the two particles can take either of the two paths (inner or outer) to reach the beamsplitters at the top of the figure. The two particles can meet if they both take the respective inner paths, in which case the two will be annihilated and photons of γ radiation will be emitted but these not detected by the particle detectors. The outgoing electrons and positrons are measured at the detectors. Experiments H1 through H4 represent four different choices of merging the respective interferometer paths or not for each of the two interferometers.

Sec. II. Even though we are using electrons and positrons, these experiments are similar to the earlier single- and two-particle experiments that relied on photons.

The electron and positron are created (for instance, via a pair-production process (Hubbell, 2006)) and shot simultaneously into the two interferometers. The electron and positron are measured either directly at detectors I_{\pm} and O_{\pm} (placed in the “inner” and “outer” paths) or at the “bright” and “dark” detectors B_{\pm} and D_{\pm} after the paths meet at a beam-splitter. The minus or plus sign in the subscript refers to either the electron (minus) or the positron (plus). Furthermore, the detectors are called “bright” and “dark” for the case with added beam-splitters because, following the discussion in Sec. II, if each Mach-Zehnder interferometer was completely on its own, the particles would always be detected at the bright detectors and never at the dark detectors.

However, the interferometers are arranged such that

there is an overlap of the inner trajectory of the electron with the inner trajectory of the positron. Note that the detectors I_{\pm} in the inner trajectories are downstream from this meeting point. If both the electron and positron take these ill-fated trajectories, then they meet and annihilate each other, since they are two halves of a particle and antiparticle pair. They create photons of γ -radiation in the process, but we assume our detectors are sensitive to electrons and positrons and not photons. As a result, if the electron and the positron meet at the intersection of their inner paths, nothing is detected at the interferometer outputs.

Let us now consider four experiments in which the electron and positrons are detected either in their inner/outer detectors (i.e., without their paths meeting) or at the bright/dark detectors (i.e., after their paths are merged).

1. Experiment H1

Let us begin by considering the experimental setup H1, which is depicted in Fig. 10:H1. In this experiment, the detectors O_-/I_- are placed directly in the two possible paths of the electron, and detectors O_+/I_+ are placed in the paths of the positron. In this case, what is the detection pattern?

Because the electron-positron pair will annihilate if the electrons and positron meet in the inner path, the pair is never observed in the two inner paths simultaneously. Hence, the detectors I_+ and I_- never click together in the same run of the experiment. Other cases of coincident detector clicks might be observed, for instance, the simultaneous detection at I_+ and O_- ; at O_+ and I_- ; or at O_+ and O_- . If this experiment is performed, then these cases, in fact, are observed with equal likelihood.

2. Experiment H2

Next, rather than measuring both the particle and the antiparticle directly, let us consider a situation in which the two paths meet at a beam-splitter for only one interferometer, and for the other interferometer the paths are measured without meeting at a beam-splitter. Let us first focus on Experiment H2, in which the right (electron) interferometer paths are combined, as depicted in Fig. 10:H2. There are two possible outcomes for the positron in the left interferometer: either the positron can be detected in the inner path or in the outer path. We can analyze these two possibilities separately.

Suppose that the positron is observed in the outer path (i.e., in detector O_+). In this case, what is the effect of the setup on the electron, i.e., what is the likelihood of observing the electron in the detectors B_- and D_- respectively? To answer this question, we observe that if the positron took the outer path, then the electron

sees a setup very similar to that of the single-particle interference, Experiment S3.

In this case, quantum physics predicts that the electron is detected only in one of the two detectors, which is the bright detector B_- . Thus, if the positron is detected at O_+ , then electrons are detected only at B_- . No electrons are observed at the “dark” detector D_- .

Next, let us suppose that the positron is observed in the inner path (i.e., in detector I_+). This time, will any electrons be observed in detector D_- ? That is, what is the likelihood of observing the electron in the detector B_- and D_- ? One may notice that now the situation for the electron is very similar to that of interaction-free measurement (Sec. II.D). The original interaction-free measurement argument included a physical obstacle blocking the path of the particle, but in this current case, the obstacle to the particle is its antiparticle meeting the particle in its path. Because of this obstacle, only electrons that take the outer path are detected. Following the reasoning of the interaction-free measurement scenario, electrons are detected at both the detectors B_- and D_- with equal likelihood.

To summarize what we know so far, if the positron is detected in the outer path, then the electrons are detected only at B_- . On the other hand, if the positron is observed in the inner path, both B_- and D_- detect electrons. Flipping the logic over and focusing on the electron detections, we observe that if electrons are detected in D_- , then the positron is certainly detected in the inner path. That is, in case detector D_- clicks, then we are certain that I_+ (and not O_+) must click as well.

3. Experiment H3

Similar detection patterns would result if the two positron-interferometer beams were combined instead and the electron beams measured without combining (Fig. 10:H3). We call this Experiment H3. That is, if the electron is detected in the outer path, then the positron paths can interfere, and all positrons are observed in the bright detector B_+ . If, alternatively, the electron takes the inner path, then the positron encounters a situation like interaction-free measurement. In this case, positrons are observed with equal likelihood in both detectors. Similar to the case of Fig. 10:H2, observing the positron in D_+ implies that the electron is detected only in detector I_- .

4. Experiment H4

In H4, the final experiment of Hardy’s paradox, the paths of both the particle and the antiparticle meet at a beam-splitter before the detectors B_+/D_+ and B_-/D_- detect the particle and the anti-particle. The outcome of Experiments H1–H3 could be predicted by applying the

concepts of single-particle interference and interaction-free measurement in a straight-forward manner. However, understanding Experiment H4 (Fig. 10:H4) requires us to carefully analyze our assumptions about the nature of reality. It is in the context of this setup that the EPR and quantum predictions differ and can be tested in real-world experiments.

B. Predictions from EPR and from Local Hidden Variables

Before describing the predictions of quantum physics, we will follow a reasoning based on the two seemingly obvious EPR assumptions of realism and locality. Recall that EPR’s first assumption is that if the value of a physical quantity can be predicted with certainty, then there is an element of reality corresponding to this quantity. EPR’s second assumption is that these elements of reality are local, i.e., they do not change instantaneously based on faraway occurrences.

What are the elements of reality in these experiments? Notice that if the electron is observed in the dark detector D_- then the positron is always observed in the inner path. Consequently, EPR would argue that there is an element of reality attached to the particle that determines which path the positron takes, at least for the case where the electron is detected in D_- . Likewise, when the positron is observed in D_+ , then it is known with certainty that the electron will be observed in the inner path. As a result, EPR would say that there is an element of reality that determines the path taken by the electron, which in this case is the inner path.

Following the reasoning of EPR, these two elements of reality exist and that these elements determine the outcomes of any experiments performed on electron-positron pairs. What does this mean for Experiment H4? According to EPR, if the positron is detected in D_+ and the electron in D_- , then the respective elements of reality determine that the electron and the positron both took the inner path. However, if both of the particles took their respective inner paths, then they must have met and annihilated in each case in which a simultaneous D_+ and D_- measurement would have been expected. Consequently, the D_+ and D_- detectors should never click simultaneously.

EPR’s second assumption, that elements of reality are local, went into the above argument as well. According to this assumption, there can be no instantaneous communication between the electron and positron because these two particles can be arbitrarily far away during the experiment. Such instantaneous communication could result in simultaneous D_+ and D_- clicks in the following seemingly contrived, but possible, scenario. Suppose that the elements of reality also keep track of whether the particle-antiparticle pair is measured at I_\pm/O_\pm or at D_\pm/B_\pm , i.e., whether the paths are combined or not.

Positrons		Electrons	
Performed measurements:		Performed measurements:	
I_+/O_+	B_+/D_+	I_-/O_-	B_-/D_-
I_+	B_+	I_-	B_-
I_+	D_+	I_-	B_-
I_+	B_+	I_-	D_-
I_+	D_+	I_-	D_-
O_+	B_+	I_-	D_-
O_+	D_+	I_-	D_-
O_+	B_+	O_-	D_-
I_+	D_+	O_-	B_-
I_+	D_+	O_-	D_-
O_+	D_+	O_-	D_-
O_+	B_+	I_-	B_-
O_+	D_+	I_-	B_-
I_+	B_+	O_-	B_-
I_+	B_+	O_-	D_-
O_+	B_+	O_-	B_-
O_+	D_+	O_-	B_-

Table III: Outcomes based on local hidden variable assumption. The two EPR assumptions are captured by the local hidden variable assumption, which states that the measurement outcomes are known beforehand for each generated electron-positron pair. In other words, the outcomes of the electron measurements are assumed to be independent of which measurements are performed on the positron and vice versa. This table lists all possible assignments of the outcomes to the four sets of measurements (inner/outer and bright/dark for the two particles) performed on the pairs. The local hidden variable argument rules out simultaneous D_-D_+ clicks as follows. The first four rows involve inner-inner detections, which result in electron-positron annihilation, and are thus ruled out. The next three assignments involve simultaneous D_-O_+ detections, which do not arise because of interaction-free measurement. Analogously, the next three assignments do not arise because of simultaneous O_-D_+ detections, which are ruled out because of interaction free measurements as well. Only the remaining six measurement outcomes are possible. These six possibilities do not include any simultaneous D_-D_+ detection.

In this case, as soon as the positron (say) reaches the far end of the interferometer and “notifies” that its path is combined, it could communicate with the electron to change its state such that it has some chance of being detected at D_- . However, if we assume locality, then this communication is impossible, provided that the measurements on each side are performed sufficiently quickly. *In summary, if we assume the existence of elements of reality and no instantaneous communication, then simultaneous D_+ and D_- clicks cannot happen.*

Although these assumptions of realism and locality were first made by EPR, the authors did not realize

the full consequences of these assumptions. It was 29 years before John Bell formalized the EPR argument in terms of local hidden variables, which led him to discover the contradiction between quantum physics and these assumptions and to design decisive tests to settle the EPR versus quantum physics debate. Bell formalized the two EPR assumptions of realism and locality into the single assumption that local hidden variables (like those presented in Sec. IV) can predict all eventual measurement outcomes.

Note that the local hidden variable assumption is stronger than the EPR assumptions. EPR assume the existence of elements of reality only if predictions with certainty are possible. In contrast, the local hidden variable assumption states that these elements always exist and underlie all measurement outcomes irrespective of whether these outcomes can be predicted with certainty. Although stronger than the EPR assumptions, the local hidden variable assumption is in the spirit of Einstein’s objections to quantum physics. Indeed, Einstein’s famous words

I, at any rate, am convinced that He [God] does not throw dice.

regarding quantum physics also indicate his inclination towards a hidden-variable approach (Ballentine, 1972). Einstein himself, after the EPR paper, moved towards phrasing the argument in terms of local hidden variables, without the ‘prediction with certainty’ condition.

The local hidden variable assumption can be stated simply in the context of the Fig. 10 setup: the measurement outcomes of each pair of interfering particles are established at their source. That is, the information about the eventual measurement outcomes is present in some possibly inaccessible (i.e., hidden) variables, which are set before the particles leave the source. These hidden variables are local in the sense that outcomes of a particle cannot change based on measurements performed on the other particle. This independence of outcomes is regarded as locality because the particles could be immensely distant when the measurements are performed on them. In this case, if the measurements are done quickly, then this independence of hidden variables could only be violated by faster-than-light communication, which is not possible.

We have already seen that the EPR assumptions rule out simultaneous D_-D_+ clicks. We now show that the local hidden variable assumption leads to the same conclusion. Because of the hidden-variable assumption, we can list all possible measurement outcomes of the electron-positron pair. Table III presents this list of possible measurement outcomes. There are a total of sixteen ($= 2 \times 2 \times 2 \times 2$) possible outcomes because each of the four total measurements (namely I_-/O_- , B_-/D_- , I_+/O_+ or B_+/D_+) can return one out of two values.

From the analysis of H1, we infer that none of the particle pairs can be assigned outcomes that include si-

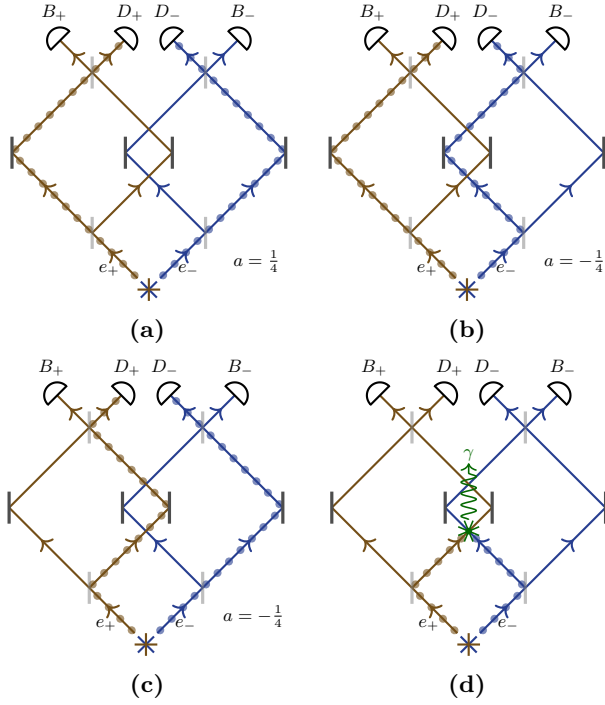


Figure 11: Calculating the probability of simultaneous clicks at D_+ and D_- using quantum theory. In quantum mechanics, the particles do not take definite paths but rather see contributions from different histories. The paths highlighted by the dots in Figures 11a–11c represent three histories that result in simultaneous clicks at D_+ and D_- . The history represented in 11a contributes an amplitude of $a = (1/\sqrt{2})^4 = 1/4$ because there are four transmission events in total at the four beamsplitters. Figures 11b and 11c each contribute an amplitude of $a = (i/\sqrt{2})^2(1/\sqrt{2})^2 = -1/4$ because of two reflections and two transmissions at the beamsplitters. The contribution from these histories should be summed and squared to obtain the likelihood $(1/4 - 1/4 - 1/4)^2 = 1/16$ of a simultaneous D_+D_- detection. Figure 11d depicts a fourth history that could have resulted in a D_+D_- detection but does not because of electron-positron annihilation.

multaneous I_-I_+ events. This rules out any pair having outcomes depicted in the first four rows of Table III. Next, we focus on Experiments H2 and H3, where the paths were combined for only one of the two interferometers. As discussed earlier in this section, the logic of interaction-free measurement rules out outcomes which include simultaneous D_-O_+ clicks or simultaneous O_-D_+ clicks. Thus, rows 5–10 of Table III are eliminated.

This means that only the six remaining measurement outcomes are possible under the local hidden variable assumption. These six outcomes can be read-off from the bottom six rows of Table III. Clearly, none of these six rows contains a simultaneous D_-D_+ outcome. *Thus, the local hidden variable assumption precludes the possibility of simultaneous D_-D_+ clicks.*

C. Quantum Predictions

While the EPR argument, or alternatively the slightly stronger local hidden variable assumption, predicts the absence of simultaneous D_+ and D_- clicks in Experiment H4, quantum physics predicts that these detectors will indeed click simultaneously in some cases. We now describe how to determine the prediction of quantum physics based on two-particle interference. Like we did in Sec. III, we calculate the prediction of quantum physics by summing over the probability amplitudes of the indistinguishable histories for the two particles and squaring the outcome.

Figs. 11a–11c depict the three possible histories of the electron and positron that result in the simultaneous clicking of D_- and D_+ . The source contributes a factor of unity to each of the histories since it shoots electron-positron pairs in the two paths in each run of the experiment. Consider the case of Fig. 11a, in which both electron and positron take outer paths. Together the electron and positron undergo a total of four transmissions at the beam-splitters, or a contribution of $1/\sqrt{2} \times 1/\sqrt{2} \times 1/\sqrt{2} \times 1/\sqrt{2} = 1/4$. The two histories depicted in Figs. 11b and 11c comprise two reflections and two transmissions. Thus, their contributions are $-1/4$ each. There is no possibility of the electron and positron both taking inner paths and being detected since such a case would have resulted in annihilation. Hence, only the remaining three (outer-outer, inner-outer and outer-inner) histories contribute. Summing over the contributions and squaring gives a $1/16$ probability of simultaneous D_+ and D_- detection.

To summarize the quantum mechanical prediction, the particles do not take definite paths but see contributions from different histories. Since there is no inner-inner history, only the other three possibilities contribute giving rise to $1/16$ likelihood of D_+D_- detections. Thus, the quantum physics predicts detectors D_+ and D_- clicking simultaneously in $1/16 = 6.25\%$ of all the possible cases. In contrast, if we assume that the outcomes of experiments are determined by local hidden variables, then there can be no cases of simultaneous detection at D_+ and D_- . Obviously, when Experiment H4 is performed in the lab, either detectors D_+ and D_- sometimes click simultaneously or they don't. This means that either we must discard the well-established rules of quantum physics or we must give up on the very intuitive idea of local elements of reality.

Years of experimental tests of Bell's original argument and of its many different variants, including the Hardy's variant, agree with the predictions of quantum physics and not with the EPR, or equivalently local hidden variable, predictions. See Sec. VI for a more detailed account of these experiments. Thus, we conclude that there are no local elements of reality or local hidden variables. This still leaves us with a choice. Either there are no elements of

reality or hidden variables that determine the outcomes of experiments before they are performed, or nature operates by means of “non-local” elements of reality or hidden variables that allow instantaneous communication.

At this stage, one may wonder if and how this strangeness of quantum physics connects with entanglement, which that we described the context of two-particles experiments in Sec. III. Indeed, the same entanglement that prevented us from describing two photons as individual entities is behind this strange contradiction between quantum physics and local hidden variables. This time, though, the entanglement is between the electron and the positron. Although the electron and the positron started out unentangled when they left their respective sources because there was no correlation between which path each of them is in. However, correlations were generated subsequently because of the possible annihilation of the electron-positron pair if both particles were in the inner paths of the interferometer. One can see that this annihilation generates correlations (i.e., mutual dependencies in the particles’ paths) as follows: if the electron arrived at its detector after taking the inner path, then we know that the positron took the other path. Similarly, if the positron took the inner path, then the electron took the outer path. This means that the particles are entangled once the histories have crossed the annihilation point. This entanglement resulted in the detection pattern that led us to conclude that quantum physics is incompatible with local hidden variables.

We can also check that indistinguishability of paths, or histories, is an important condition for observing a departure from local hidden variable predictions. In our calculations, we obtained the detection probabilities by adding amplitudes from three indistinguishable histories. If these histories are made distinguishable by adding NDDs in the paths (in analogy to Experiments S5, S6 and E5 before the particles arrive at their respective downstream beam-splitters, then the detection probabilities change. Although the quantum calculations still predict a small probability of simultaneous detection in D_+ and D_- , this probability is not enough to rule out a local hidden variable description if the NDDs are accounted for in the calculations, because the results of the other experiments change as well when NDDs are introduced, since they disturb the interference. Remarkably, in the presence of NDDs, quantum physics and local hidden variable theories predict identical probabilities for simultaneous detection in each of the four possible measurement scenarios. We will see in Sec. VI that this requirement of indistinguishability poses a key challenge in experimentally ruling out local hidden variables and thus verifying quantum physics.

D. Nonlocal Hidden Variables and the Pilot-Wave Theory

As described above, there are two resolutions to Hardy’s paradox: either nature does not operate by means of hidden variables, or these hidden variables are not local. While most quantum physicists lean towards giving up on the idea of hidden variables entirely (Schlosshauer *et al.*, 2013), it is possible to construct non-local hidden variable theories that agree with all quantum predictions. The most popular such non-local hidden variable theory is the pilot wave theory, which was introduced by Louis de Broglie in 1927 and further developed by David Bohm in 1952. The pilot-wave theory takes both the wave and particle aspects of quantum physics literally: in addition to the particles, there is also a wave that is present everywhere and can interact with the particles. To match quantum predictions with those of the pilot-wave model, we have to discard locality and necessarily assume that the pilot wave admits nonlocal influences, i.e., it can be influenced instantaneously by distant operations. Specifically, the measurement made on one particle belonging to an entangled pair instantaneously changes how the pilot wave acts on the other, possibly distant, particle.

Although the pilot wave can change instantaneously based on distant events, it still cannot be used for faster-than-light communication in practice. This is because we do not have access to the exact position and location of each particle as these will inevitably be shrouded in randomness. It turns out that after observing many random events, the average outcomes remain the same independently of any distant events.

The pilot-wave theory explains not just the outcomes of experiments H1–H4, but can account for all quantum phenomena. That is, the predictions of the pilot-wave theory are identical to those of quantum physics. Hence, it is fair to say that the pilot-wave theory is as correct as quantum physics. Although we might prefer one over the other as a matter of taste, we cannot tell the difference between their eventual predictions.

E. Summary

We conclude this section with a summary. Bell’s theorem indicates a deep contradiction between quantum physics and EPR’s seemingly obvious assumptions about reality. Surprisingly, experiments demonstrate quantum physics to be correct and rule out the intuitively appealing possibility of local hidden variables. This implies that measurement outcomes are either not predetermined at all, or they are determined by nonlocal hidden variables, such as Bohm’s pilot wave model.

VI. EXPERIMENTAL WORK

No theory of physics is meaningful without rigorous verification and testing (Ede and Cormack, 2016; Oxford, 2018). Throughout history, experimentation has played a key role in shaping and supporting the predictions of physical theories. In a theory as radical and counter-intuitive as quantum physics, experimentation has been instrumental not only its validation but also its broad acceptance (Ede and Cormack, 2016). In this section we discuss some of the key experiments that have justified our support of quantum physics.

To motivate the challenges of quantum experiments, we first discuss the concept of quantum decoherence, which destroys wave-particle duality by removing indistinguishability between histories. We then introduce some properties of photons that enable real-world experiments using these particles. Next, in support of the reasoning of Secs. II–V, we discuss experiments that demonstrate wave-particle duality and the rejection of local hidden variables, the latter via loophole-free tests of Bell’s theorem. We conclude the section with a short discussion on the technological applications of quantum physics, with focus on information processing.

A. Indistinguishability and the environment

Accurate and precise testing of physical theories that lie within the boundaries of day-to-day intuition, such as those that are underpinned by local hidden variables, can be challenging. Thus, devising and executing an experiment to test quantum physics, which counters our cherished assumptions about the nature of reality, requires an extra degree of ambition and vigilance. Beyond this, an experimental quantum physicist needs to proceed with care due to the fact that quantum systems are easily modified by their environment (Zurek, 1991). This is because it takes only one external, unaccounted, particle to reveal information about a quantum particle that is under test and hence destroy the indistinguishability of its histories.

Specifically, recall experiments involving NDDs (S5, S6, and E5) and the corresponding experiments in which the NDDs are absent (S3 and E2). These experiments show that quantum interference between histories is observed only when it is not possible to determine which history the photons actually took. That is, as soon as some other system obtained information about the histories that the photons took, then the indistinguishability was lost and the interference disappeared.

In these example, the NDD’s influence is interacting with the photons and destroying their interference, but such influences from the environment of an experiment are common and hard to avoid. That is, as soon as the environment interacts with the quantum particles and

“sees” the particles in one history, the indistinguishability is lost. Remarkably, even if the experimenter does not measure the environment, the different paths are rendered distinguishable. These unwanted interactions with the environment, which are referred to as decoherence, are one important reason why it is hard to observe quantum effects.

This loss of indistinguishability from the environment provides a resolution to the famous thought experiment of Schrödinger’s cat (Schrödinger, 1935), which offers the possibility of a cat to be in a superposition of dead and alive states. The existence of the cat in a superposition neglects to account for the fact that the cat will continuously interact with the environment, such as atoms of air bouncing off from the cat or all-pervasive microwave-frequency photons from the early universe (Durrer, 2008). This means that, in practice, the cat will only be observed to be either dead or alive, and no quantum strangeness will be observed.

Although the environment can play a negative role but preventing us from observing quantum strangeness in the laboratory, modern experiments meticulously isolate the quantum system from the environment and are able to observe wave-particle duality as described below in Sec. VI.C.

Note that interaction-free measurements are routinely observed in quantum optics laboratories. In fact, they are used to calibrate interferometers. An experiment that is conceptually identical to that of Fig. 6 has been performed (Kwiat *et al.*, 1995). Furthermore, experiments on non-destructive detection have also been completed (Grangier *et al.*, 1998), with recent work, e.g. (Reiserer *et al.*, 2013), focusing on developing non-destructive detectors for quantum information processing applications (see Sec. F).

B. Photons and their Properties

Arguably the most common particle that is used when testing quantum principles is the photon (Shadbolt *et al.*, 2014; Zeilinger, 2010). Referring to the Hardy experiment of Sec. V, one should realize that although electron-positron pairs may be produced, they are done so in dedicated, expensive, and bulky particle accelerators that most scientists do not have at their disposal. In contrast, photons can be produced and manipulated fairly easily, and they interact relatively weakly with each other and the environment. Photons have been employed in many single and two-particle interference experiments that are similar to those described in the single- and two-particles experiments, i.e., Experiments S1–S6, E1–E5, and H1–H4.

Polarization is one of the most widely-employed properties of a photon in quantum experiments (Lundeen and Steinberg, 2009; Shadbolt *et al.*, 2014; Torgerson *et al.*, 1995; Zeilinger, 2010). This is partially due to the avail-

ability and quality of polarization-dependent optics (Bass *et al.*, 2009). From the classical perspective of light as an electromagnetic field, the polarization denotes the direction of oscillation of the electric field-component of light (Hecht, 2016). When light is described quantum-mechanically in terms of photons, a similar definition applies, but with the possibility of the polarization of light being in superposition (Mandel and Wolf, 1995). For example, a photon can be polarized at 45° (with respect to some reference, e.g. a laboratory bench), which can be equally described by it being in an indistinguishable superposition of being polarized at 0° and 90° .

Other widely-employed properties of photons include their path (or spatial properties), their energies (or frequencies, wavelengths), or their time of arrival. The experiments described in the previous section have focussed on the path properties of photons. The path properties have been used for loophole-free Bell tests (see, e.g., the Delft experiment of Sec. VI.E) while their energies and time of arrival are used for quantum applications (e.g., quantum communications of Sec. VI.F).

C. History of Wave-Particle Duality

Before the development of quantum physics, it was widely accepted that light was best described as a wave (Ede and Cormack, 2016). Although Newton argued in favor of the interpretation of light as consisting of particles in the 18th century (Newton, 1704), a famous experiment by Young in 1801 gave experimental support to Huygens' initial idea that light is a wave (Huygens, 1690; Young, 1804). Drawing from his experience with sound, Young performed an experiment that involved directing a beam of light at a pair of thin slits (strictly speaking, a light beam was split by two sides of a thin paper slip). The light was then directed to a screen. This lit up the screen with periodic bright and dark bands, which is a signature of wave interference. His experiment is analogous to the setup of Experiment S4, except Young's experiment allowed (classical) light to traverse several different paths, each of a different length. The wave-model of light was upheld until Planck mathematically showed that light must be absorbed and emitted in quanta in order to explain the spectra (the range of light-colors) produced by radiation from a hot body (Planck, 1900). This idea was followed-up by Einstein, who proposed that the description of light as particles explained the energies and numbers of electrons that are ejected from a metal when bombarded with light, referred to as the photoelectric effect. This earned him a Nobel prize in 1921 (Einstein, 1905b). Experiments by Lenard supported Einstein's explanation of the photoelectric effect in 1907, even though his initial intent was to disprove its existence (Lenard, 1902).

A 1986 experiment by Grangier, Roger, and Aspect,

which used set-ups that are conceptually identical to Experiments S1, S3, and S4, clearly illustrated wave-particle duality of single photons (Grangier *et al.*, 1986). A single photon was generated by first detecting one half of a photon pair that was emitted from a cloud of Calcium atoms. Consistent with the predictions of Experiment S1, the single photon never led to two detectors (D_1 and D_2) responding simultaneously. The only exception was a small rate of such coincident detections due to experimental noise. This demonstrated the particle character of the photons. To demonstrate their wave character, Grangier and colleagues sent single photons into a Mach-Zehnder interferometer. Varying its relative path length, they observed a detection rate (at D_1 and D_2) that varied sinusoidally on a near-zero background, consistent with wave interference.

As we introduced in Sec. II, modern physics now tells us that a photon possesses both wave-like and particle-like characteristics. The wave-like properties are supported by the single-photon version of Young's experiment of Fig. S3, while particle-like properties are revealed by photoelectric experiments, single photon detectors, and Experiments S1 and S2.

In analogy to light, support for a wave-particle duality of matter was initiated by de Broglie (De Broglie, 1924). There was previously strong evidence that matter was particle-like, beginning with Dalton's 19th century deduction of the existence of atoms from chemistry (Dalton, 1806) to Einstein's Brownian motion explanations of 1905 (Einstein, 1905a). The first experimental support for De Broglie's hypothesis that matter can also behave like waves was provided by Davisson and Germer in 1921, who fired electrons at crystalline nickel (Davisson and Germer, 1928). The interaction of the electrons with the periodic array of atoms which constitute the nickel produced alternating bands of high- and low-density electrons in a similar spirit as to how Young's photons interacted with slits to produce periodic bands of light. The first matter-wave experiments that probed the nature of single individual quanta were pioneered by Rauch in 1974, who employed individual neutrons that scattered from silicon crystal plates (Rauch *et al.*, 1974). Since then, interference of large molecules such as C_{60} (Arndt *et al.*, 1999) and biologically-significant neurotransmitters has been shown (Geyer *et al.*, 2016), with a path towards interference of large bio-matter such as proteins (Geyer *et al.*, 2016). In all of these experiments, many steps were taken to ensure indistinguishability of the different paths, e.g. by reducing the amount of radiation from large molecules or the presence of background gases. Indeed, modern quantum physics endorses the existence of matter wave-particle duality in analogy with that of light.

Beyond quantum duality, a lot of recent scientific interest has focused on the generation of matter systems that exhibit, in some cases mesoscopic (see for example Appel *et al.*, 2009; Gross *et al.*, 2012; McConnell *et al.*, 2015;

Zarkeshian *et al.*, 2017), quantum entanglement in order to further probe where the rules of classical physics end and those of quantum physics begin, and also to develop new applications.

D. Bell Tests and Hardy's Experiment

As discussed in Sec. V, the conclusions that are drawn from Hardy's thought experiment are in conflict with the assumptions of EPR. If the electron and positron are governed by local elements of reality, then their annihilation should *never* result in a detection event (i.e. detectors $D_{+/-}$ of Fig. H4 should not fire). A (healthy) skeptic may ask about the role of experimental imperfections in this context. For example, consider the fact that single photon detectors may occasionally produce a (supposed) detection result, even if no photons are sent to the detector (Eisaman *et al.*, 2011). This is called a 'dark count'. So the question arises: what amount of experimental errors of this kind is tolerable to make the claim that Hardy's experiment or other similar experiments genuinely refute the existence of local elements of reality? This question can be answered by following the approach of Bell (Bell, 1964). The so-called Bell test, and more experimentally-friendly versions thereof, such as that of Clauser, Horne, Shimony and Holt (CHSH) (Clauser *et al.*, 1969), importantly rely not on a single experimental observation, but on the collection of measurements from multiple runs of an experiment. This ensures that any conclusions are not formed based on only a single measurement, of which could be impacted by experimental imperfections.

Bell's theorem begins with first defining a mathematical relation that is composed of a set of average values of measurement outcomes of an experiment. Next this relation is bounded with the assumption that experimental outcomes are predicted by local hidden variables, resulting in a (Bell) inequality (Bell, 1964). For example, consider the table of outcomes based on local hidden variables, Table II (III), for the entangled system (Hardy experiment) of Fig. 7(H4). When an experiment produces results that contradict the assumptions of EPR, the evaluated relation will surpass the bound, which is referred to as a 'violation' of a Bell inequality. Furthermore, a standard deviation (which is determined from each measurement) is used to determine the statistical uncertainty of the amount of violation. Bell tests are commonly performed using pairs of entangled particles (e.g. photons, atoms, neutrons, etc), in which one out of a set of two possible measurements is performed on each particle. For a Bell test using Hardy's experiment, these correspond to either the I_{+}/O_{+} or the B_{+}/D_{+} experiment for the positron, and analogously for the electron. The results of these measurements, when combined appropriately, can demonstrate a violation of a Bell inequality (Irvine *et al.*, 2005). Thus, to answer the skeptic, a Bell inequality establishes a limit to the

amount of experimental uncertainty that can be tolerated in an experiment in order for it to conclusively disprove the existence of local hidden variables.

Bell's theorem was tested for the first time in 1972, in an experiment by Freedman and Clauser (Freedman and Clauser, 1972). They used pairs of polarization-entangled photons that were emitted from individual calcium atoms. Yet, despite a violation (and the impressive fact that they used duct tape and spare parts to construct their experiment), there was suspicion that their results could be explained from experimental imperfections. Added doubt was cast by a concurrent experiment by Holt and Pipkin that did not violate a Bell inequality (Holt, 1973; Pipkin, 1979). Stronger proof was provided in the early 1980s by a series of experiments by Aspect and colleagues (Aspect *et al.*, 1982a, 1981, 1982b). One of their achievements was to improve the calcium-based photon source of Freedman and Clauser to efficiently produce stable polarization-entangled photons (recall from Sec. C that they also used this source to generate single photons). As a result, good statistical accuracy was achieved, leading to a violation of a Bell inequality by five standard deviations (Aspect *et al.*, 1982a).

E. Closing Loopholes in Bell Tests

Another achievement of Aspect's 1982 experiment was to address a loophole that was present in all previous experiments (Aspect *et al.*, 1982a). A loophole refers to the possibility of explaining the violation of a Bell inequality using a local hidden variable model, and is possible if the assumptions that underpin the formulation of the Bell inequality are not met in an experiment. In order to establish *locality* conditions for a Bell test using two particles, an experimenter must ensure that the choice of measurement that is performed on the first particle cannot (even in principle) be communicated to the second particle (and vice versa) (Aspect *et al.*, 1982a; Larsson, 2014; Weihs *et al.*, 1998). To envision this, one may imagine that a signal traveling at the speed of light (nothing can travel faster according to the theory of relativity) is emitted by the measurement apparatus for the first particle as soon as the choice of measurement has been made. If this signal were to reach the second particle, then it could, in principle, alter the behavior of the second particle such that its measurement could result in a violation of Bell's theorem. The magnitude of the speed of light implies that for a 'locality loophole-free' test of a Bell inequality the measurements must be chosen very quickly.

Aspect's experiment was the first to address the locality loophole by independently and rapidly choosing measurement settings after the photons were emitted (Aspect *et al.*, 1982a). Nonetheless, his experiments involved manipulating the measurement settings by using a predictable switching mechanism (specifically he deflected

photons by using periodically-driven sound waves), which was not a completely satisfactory solution for closing the loophole since, in principle, the source might have somehow “learned” that the settings were predictable, and adjusted the particles’ hidden variables accordingly to still violate a Bell inequality (see the discussion of the “fair sampling” loophole below). It was not until 1998 that Weihs and colleagues definitively closed the locality loophole (Weihs *et al.*, 1998). This was achieved by separating two entangled photons by 400 meters (to increase the time available for switching measurements) and by using a truly-random number generator to decide the measurement settings. In fact, their random numbers were generated from a device that is conceptually identical to the setup of Fig. S1: the detection of a weak pulse of light in detector D_1 and D_2 corresponded to the generation of a bit with value 0 and 1 respectively. Their exploitation of quantum randomness avoided the need of using classical pseudo-randomness (which is predictable, in principle). The measurement was performed using a robust and high-rate source of polarization-entangled photons that were generated by a process called spontaneous parametric down-conversion, which was well-mastered by the time of the experiment.

Beyond locality, the so-called *fair sampling* loophole must also be closed (Larsson, 2014; Rowe *et al.*, 2001). This refers to the requirement that the particles which are detected in a Bell test should accurately represent all the particles that are generated during the course of the experiment. If particles can somehow decide to not be detected depending on the measurement settings, then the ones that are detected can again violate a Bell inequality. For typical Bell inequalities a total detection efficiency of over 2/3 (assuming no other experimental noise) is needed to close this loophole (Clauser and Horne, 1974; Eberhard, 1993). It is not trivial to achieve such high detection efficiency for photons, especially since all kinds of loss (including propagation losses, losses in experimental apparatus, etc.) have to be taken into account. For example, in the experiment by Weihs *et al.* the detection efficiency was only a few percent (Weihs *et al.*, 1998).

The fair sampling loophole was first closed by Rowe, Wineland (who won the Nobel Prize in 2012), and others in 2000 (Rowe *et al.*, 2001). This was achieved by using two charged Beryllium ions that were ‘trapped’ (i.e. held roughly stationary) using time-varying electric fields. The magnetic properties of each ion, specifically their total angular momentum, was entangled using laser-induced excitation and the electric force that each ion experienced by the other. Measurements were accomplished by exciting each ion with a laser and detecting the photons that are emitted when the ion decays. The key to closing the loophole lies in the fact that the experiment made use of a ‘cycling transition’. This meant that after a photon emission the ion returned to its original (entangled) state, allowing it to be immediately re-excited and

emit another photon. This made it possible to determine the state of the ions reliably for every repetition of the experiment, even though many of the emitted photons were lost. This allowed the violation of a CHSH-Bell inequality by more than 8 standard deviations. This result highlights the advantage such a system has over the photon-photon entanglement experiments that require the irreversible destruction of entanglement by (lossy) single photon detectors.

It took until a 2015 experiment at Delft to close both loopholes simultaneously (Hensen *et al.*, 2015). This experiment used atom-like systems, specifically impurities in diamond, and a clever method to entangle them. Two individual impurities, separated by 1.3 kilometers, were excited using a laser such that, with some probability, they could each emit a photon towards a beam-splitter that is situated halfway between the impurities. Due to the fact that each of these photons are indistinguishable, a single photon detection event at the output of the beam-splitter ensured that the impurities became entangled. This so-called ‘event-ready’ scheme allowed the fair-sampling loophole to be closed because the detection event heralded the situation when both impurities had become (remotely) entangled (Simon and Irvine, 2003; Żukowski *et al.*, 1993). Similar to Rowe *et al.*, a laser-probed cycling transition was used to perform the required measurements, while the separation of the impurities ensured enough time to enforce locality. One major challenge in this experiment were very low detection rates. Only 245 events were recorded in total, which resulted in a violation of the CHSH-Bell inequality by only 2 standard deviations, which might have induced a skeptic to doubt the conclusion on purely statistical grounds. However, soon after, experiments in Vienna and Boulder also closed both loopholes (Giustina *et al.*, 2015; Shalm *et al.*, 2015). These experiments utilized polarization-entangled photon pairs (perhaps to no surprise) that were distributed over 30 and more than 100 meters, and featured violations of a CH-Eberhard-type inequality (another version of Bell inequalities) by 11 and 7 standard deviations, respectively, removing any potentially lingering doubts based on statistics. These experiments utilized rapidly-switchable polarization-selective filters, high-speed electronics, and exploited low-loss superconducting wire-based single-photon detectors. After these three experiments, there is now broad consensus in the scientific community that local hidden variables have been definitively ruled out by experiments.

F. Applications

Besides their fundamental interest, quantum phenomena are also important for practical applications (see e.g. (Bardeen and Brattain, 1948; Gill, 2017; Nielsen and Chuang, 2010; Schawlow and Townes, 1958)). One phenomenon that is important for the semiconductor industry

is quantum tunneling (Bardeen and Brattain, 1948; Hund, 1927), in which an electron can be in a superposition of being transmitted through a charged barrier or not. Global positioning systems rely on clocks which ‘tick’ at a rate defined by the quantized energy states of atoms (Essen and Parry, 1955; USAF, 2018).

More directly related to the topics discussed in this manuscript, quantum information processing aims to exploit wave-particle duality and entanglement to process information in novel ways (Nielsen and Chuang, 2010). This idea was first suggested by Manin (1980) and separately Feynman (1982), who envisioned using certain controlled quantum systems to simulate complex quantum dynamics and interactions (Feynman, 1982; Manin Yu, 1980). By the mid 1980s, scientists were beginning to consider the possibility of quantum systems to be able to process and manipulate information in ways that classical systems cannot. For example, in 1985, Deutsch devised a quantum algorithm that, based on the properties of entanglement, is exponentially faster than any possible classical algorithm (Deutsch, 1985; Nielsen, 2018). In 1984, Bennett and Brassard showed that, by exploiting wave-particle duality, a secret key can be established between two parties if information is encoded into a single quantum particle (e.g. by encoding bits using the polarization of a photon) (Bennett and Brassard, 1984). Furthermore in 1993, scientists showed how to transmit quantum (information) states (potentially over long distances) using entanglement (Bennett *et al.*, 1993), using a method known as quantum teleportation, with the first experimental demonstration following in 1997 (Bouwmeester *et al.*, 1997). These discoveries spawned the research fields of quantum computing, quantum cryptography, and quantum communication respectively, which are all sub-fields of quantum information science (Nielsen and Chuang, 2010). Recently, individual atoms in crystals have been shown to be promising as sensors of external stimuli (e.g. heat, electric and magnetic fields), spawning the field of quantum sensing (Degen *et al.*, 2017). These discoveries have produced a multitude of experiments, including earth-to-satellite quantum communications (Billings, 2018), and have resulted in industrial efforts as well as commercially-available products, e.g. quantum random number generators (Gill, 2017).

VII. DISCUSSION

In terms of the disparity between the levels of our practical and philosophical understanding of our theories about the natural world, quantum physics stands out as arguably the most stark example. The theory has resisted attempts at being intuitively internalized by even its discoverers and practicing experts, let alone by the wider human culture. But some creative physicists and philosophers of the last few decades have demonstrated

that an intuitive understanding of quantum physics may not be altogether impossible.

In this manuscript, we followed the lead of these pioneers and aimed to describe two key surprises of quantum physics via simple thought experiments that are based on interference. First, we described how the interference of a single particle displays the properties of both waves and particles, yet is somehow different from either. The case of interaction-free measurement, wherein a photon detects the presence of an obstacle without actually interacting with it, exemplifies quantum weirdness in the single-particle case.

Although these single-particle phenomena seem counter-intuitive, it is possible to explain them using the seemingly intuitive concepts of local hidden variables. The second key surprise of quantum physics is that the outcomes of two-particle interference are incompatible with the notion of local hidden variables.

We followed Hardy’s approach (Hardy, 1992) to Bell’s theorem, and showed how the expected outcomes predicted by quantum interference are fundamentally at odds with the outcomes expected from a local hidden variable approach. Although it is possible to explain the outcomes of quantum physics using hidden variables, these explanations inevitably involve instantaneous communication between particles.

These two surprising aspects of quantum physics have been verified over decades of experimental effort, beginning with the 1982 experiments of Aspect, Roger and Grangier (Aspect *et al.*, 1982b) that demonstrate wave-particle duality of single particle, followed by the first tests of Bell’s version of quantum weirdness, and culminating in recent loophole-free tests of the second key surprise.

Looking towards the future, let us mention several significant open questions involving quantum physics. They are all questions regarding its ultimate domain of applicability and relevance.

Quantum physics is an essential part of the framework for the standard model of particle physics, which describes electromagnetism as well as the weak and strong interactions. Despite the efforts of many physicists over several decades, it is not clear yet whether quantum principles also apply to gravity (Smolin, 2008).

A different, if potentially related, question is whether quantum principles apply to macroscopic objects, or whether there is some scale of mass, size or complexity where they cease to be valid. New experiments are constantly being developed to extend the domain in which quantum effects have been demonstrated (Arndt and Hornberger, 2014). So far quantum physics has withstood all of these challenges.

A fascinating open question is to what extent quantum principles play a role in biological systems (Lambert *et al.*, 2013; Mohseni *et al.*, 2014). Many scientists are investigating potential roles for quantum effects in photosynthesis (Engel *et al.*, 2007; Romero *et al.*, 2014) and in

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